Fall 10, College Algebra I Test 2 Solutions Howard University Mathematics
Math 006-19 Sitaraman 10/4/2010

1.(a) Simplify the following and write answer with positive exponents: $\left(\frac{8}{27}\right)^{\frac{2}{3}}$

(b) Rationalize the denominator of $\frac{13}{2-\sqrt{3}}$

Soln:

$$(a) \quad \left(\frac{8}{27}\right)^{\frac{2}{3}} = 8^{\frac{2}{3}}/27^{\frac{2}{3}} = \frac{(2^3)^{\frac{2}{3}}}{(3^3)^{\frac{2}{3}}} = \frac{2^{3(\frac{2}{3})}}{3^{3(\frac{2}{3})}} = 2^2/3^2 = 4/9.$$

(b) Multiply above and below by the conjugate of the denominator, namely $2+\sqrt{3}$. You get

$$\frac{13}{2-\sqrt{3}} = \frac{13(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})} = \frac{26+13\sqrt{3}}{2^2-(\sqrt{3})^2} = (26+13\sqrt{3})/(4-3) = 26+13\sqrt{3}.$$

2. Solve : $x^2 - 2x + 5 = x^2 + x + 10$.

Soln: Bringing all to one side, we get $x^2-2x+5-x^2-x-10 = x^2-x^2-2x-x+5-10 = -3x-5 = 0$. Solving we get $-3x = 5 \Rightarrow x = -5/3$.

3. (20 points) With a discount of 20 percent a dress costs \$642.40.[i.e, you reduce the original price by subtracting 20 percent of it from itself].What is the original price?

Soln:Let P be original price. Then 20 percent of P is $0.2P$ [because 20/100=0.2]. So we get $P - 0.2P = 642.40$. This gives $0.8P = 642.40$ which means $P = \frac{642.40}{0.8} = 803$. So original price is \$803.

4.(20 points) Solve $x^2 + 4x - 3 = 0$ using (a) completing the square (b) quadratic formula

Soln:(a)$x^2+4x-3=0 \Rightarrow x^2+4x=3 \Rightarrow x^2+4x+2^2 = 3+2^2 \Rightarrow (x+2)^2 = 7 \Rightarrow x+2 = \pm\sqrt{7} \Rightarrow x = -2\pm\sqrt{7}$.

(b) Using quadratic formula, $a=1, b=4, c=-3$. We get $x = \dfrac{-4\pm\sqrt{4^2-4(1)(-3)}}{2(1)}$

$= \frac{-4\pm\sqrt{28}}{2} = \frac{-4\pm\sqrt{(4)(7)}}{2} = \frac{-4\pm2\sqrt{7}}{2} = \frac{-4}{2} \pm \frac{2\sqrt{7}}{2} = -2\pm\sqrt{7}$.

5. The height of a projectile after t seconds is given by $s = 96 + 80t - 16t^2$. After how many seconds does the projectile hit the ground? [i.e, when is $s = 0$?]

Soln: We want $s = 96 + 80t - 16t^2 = 0$. Dividing all by 16 we get $6 + 5t - t^2 = 0$. Taking negative of both sides and writing it in standard format we get $t^2 - 5t - 6 = 0$. This factors as $(t-6)(t+1) = 0$ which gives $t-6 = 0$ or $t+1 = 0$. That is $t = 6$ or $t = -1$. Since time is positive 6 seconds is the answer.

6. Solve for t : $\sqrt{2t+3} = 3$.

Soln: Squaring both sides, we get $(\sqrt{2t+3})^2 = 3^2 \Rightarrow 2t+3 = 9 \Rightarrow 2t = 6 \Rightarrow t = 3$. Checking in orginal equation, we get $\sqrt{2(3)+3} = \sqrt{9} = 3$ which is correct.

7. Simplify and then solve the following quadratic equation: $5x + 2 = 13 - (x+3)^2$.

Soln: Bringing all to left side we have $(x+3)^2 + 5x + 2 - 13 = 0 \Rightarrow (x^2 + 6x + 3^2) + 5x + 2 - 13 = 0$. This gives $x^2 + 11x - 2 = 0$. Solving using quadratic formula, $[a = 1, b = 11, c = -2]$ we get $x = \frac{-11\pm\sqrt{11^2-4(1)(-2)}}{2} = \frac{-11\pm\sqrt{129}}{2}$.

8. Solve $-1 \leq (3x+1) \leq 1$ and write answer in interval form.

Soln: The aim is to get x by itself. First subtract 1 from all sides: $-1-1 \leq 3x+1-1 \leq 1-1$ which is $-2 \leq 3x \leq 0$. Now divide all by 3. You get $-2/3 \leq x \leq 0$. This can be written in interval form as $[\frac{-2}{3}, 0]$.

9. [Challenge problem, 20 points extra credit] Show that if two consecutive natural numbers form the legs of a right triangle, then the hypotenuse will be an odd number.[It is not enough to check a few cases. You have to prove for ANY two consecutive numbers].

Soln:Note: It is possible that sums of squares of two consecutive numbers is not the square of a natural number. example: $2^2 + 3^2 = 13$ and 13 is not the square of a natural number. So the hypotenuse in this case has length $\sqrt{13}$ which is not natural. In this case it is meaningless to say that it is not an odd number, but I have given you full credit if you said the problem is wrong because of this. I should have said in the problem that the hypotenuse must also be a natural number.

You can say that out of two consecutuve numbers, one has to be an odd number, and so the sum of the squares is odd because the square of an even number is even and square of odd number is odd and when you add even and odd you get an odd number. So the square of hypotenuse is odd. But the square of an even number cannot be an odd number, so the hypotenuse must be an odd number.

A quicker way to say this is (and it includes proof that square of even is even and square of odd is odd and sum of even and odd is odd):

Let n be the first number. Then the next one is $n+1$. Adding their squares we get: $n^2 + (n+1)^2 = n^2 + n^2 + 2n + 1 = 2n^2 + 2n + 1 = 2(n^2 + n) + 1$. This is the square of the hypotenuse. It is an odd natural number because it is 1 plus an even number, because $2(n^2 + n)$ is an even number [it has 2 as a factor]. The square of an even number cannot be an odd number, so the hypotenuse must be an odd number.