Fall 10, College Algebra I Test 1 Howard University Mathematics
Math 006-19 Sitaraman 9/13/2010

Instructions:
PLEASE PROVIDE STEP BY STEP EXPLANATIONS
Time Limit 45 minutes
Please read the questions carefully before answering
Each problem 10 points unless otherwise stated.
Total 120 points including 20 points extra credit for challenge problem.

1. Simplify the following and write answer with positive exponents:
(a) $\frac{(3^{-10})^{24}}{3^{17}}$ (b) $(xyz)^{30}x^3y^5z^6$

Soln: (a) $\frac{(3^{-10})^2 4}{3^{17}} = \frac{3^{-10\times 24}}{3^{17}} = 3^{-240-17} = 3^{-257} = \frac{1}{3^{257}}$ because we have to write it with positive exponents.
(b) $(xyz)^{30}x^3y^5z^6 = x^{30}y^{30}z^{30}x^3y^5z^6$
$= x^{30+3}y^{30+5}z^{30+6} = x^{33}y^{35}z^{36}$.

2. (a) Write 0.010345 in scientific notation (b) Write 2.5×10^6 in decimals.

Soln:(a) 1.0345×10^{-2} because you have to move the decimal 2 places to the left. (b) 2500000

3. Is it possible for a right angled triangle to have lengths 22, 20 and 9? Use Pythagoras theorem. [Hint: Hypotenuse is always the longest side, IF it were a right angled triangle].

Soln: If it were a right angled triangle, we would have $22^2 = 20^2 + 9^2$ which gives $484 = 400 + 81$ but that is not true. So this is NOT a right angled triangle.

4.(20 points)
(a) Simplify after multiplication: $[(x^2 - 3x + 1)(x - 6)] - [x^3 - 14x^2]$
(b)Factor by grouping and looking for common terms: $x^3 + x^2 + x + 1$.

Soln:(a) $[(x^2 - 3x + 1)(x - 6)] - [x^3 - 14x^2] = [x^3 - 3x^2 + x - 6x^2 + 18x - 6] - [x^3 - 14x^2] = x^3 - 9x^2 + 19x - 6 - x^3 + 14x^2 = 5x^2 + 19x - 6$
(b)$x^3 + x^2 + x + 1 = (x^3 + x^2) + (x + 1) = x^2(x + 1) + 1(x + 1) = (x^2 + 1)(x + 1)$.
Note that $x^2 + 1$ cannot be factored further.

5. Factor completely $3x^2 - 10x - 25$.

Soln: We have 3(-25) = -75. So we try to find two numbers A,B so that AB = -75 and A+B = -10. We find that -15 and 5 work. So we write the middle term as a sum to get $3x^2 - 10x - 25 = 3x^2 - 15x + 5x - 25$. We group two terms at a time and then factor out common terms: $(3x^2 - 15x) + (5x - 25) = 3x(x-5) - 5(x-5) = (3x+5)(x-5)$. Check that answer is right by multiplying.

6. (a).(4 points)If you were to expand $(2x+3y)^3$ using the formula $(A+B)^3 = A^3 + 3A^2B + 3AB^2 + B^3$ what would A and B be?

(b) (4 points) Write down $A^3, B^3, 3A^2B$ and $3AB^2$ with what you get for A and B in part (a).

(c) (2 points) Add the expressions in part (b) to get $(2x+3y)^3$.

Soln:(b) $A = 2x$, $B = 3y$. (b) $A^3 = 8x^3, B^3 = 27y^3, 3A^2B = 3(4x^2)(3y) = 36x^2y, 3AB^2 = 3(2x)(3y)^2 = 3(2x)(9y^2) = 54xy^2$. (c) $(2x+3y)^2 = 8x^3 + 36x^2y + 54xy^2 + 27y^3$.

7. (20 points) Add and simplify as much as possible : $\frac{2x-3}{x^2+8x+7} - \frac{x-2}{(x+1)^2}$.

Soln:First factor the denominator of the first fraction (second fraction's denominator is already factored). We get $\frac{2x-3}{(x+7)(x+1)} - \frac{x-2}{(x+1)^2}$

Now the LCM is $(x+1)^2(x+7)$. Multiply each fraction by suitable terms above and below so that you can write both with same denominator (the LCM). We get $\frac{(2x-3)(x+1)}{(x+7)(x+1)^2} - \frac{(x-2)(x+7)}{(x+1)^2(x+7)}$

Now add the numerators and simplify:

$$\frac{(2x-3)(x+1)-(x-2)(x+7)}{(x+7)(x+1)^2} = \frac{(2x^2-x-3)-(x^2+5x-14)}{(x+7)(x+1)^2}$$

$$= \frac{(x^2-6x+11)}{(x+7)(x+1)^2}.$$

The numerator cannot be factorized further, so this is the final answer.

8. Divide $7x^3 - x^2 + 3x + 5$ by $x - 2$ and write the quotient and remainder.

Soln:

$$\begin{array}{c|cccc} 2 & 7 & -1 & 3 & 5 \\ & 0 & 14 & 26 & 58 \\ \hline & 7 & 13 & 29 & 63 \end{array}$$

The quotient is $7x^2 + 13x + 29$. The remainder is 63. You can check the answer: Multiply the quotient by $x - 2$ and add the remainder to see if you get $7x^3 - x^2 + 3x + 5$.

9. [Challenge problem, 20 points extra credit] Show that $n^2 - 1$ is always a multiple of 3 as long as the value of n is NOT a multiple of 3. For example, $1^1 - 1 = 0, 2^2 - 1 = 3, 4^2 - 1 = 15, 5^2 - 1 = 24 \ldots$, all divisible by 3 but $3^2 - 1 = 8, 6^2 - 1 = 35, 9^2 - 1 = 80, \ldots$ none divisible by 3. You have to explain why this is always the case, especially why $1^2 - 1, 2^2 - 1, 4^2 - 1, 5^2 - 1 \ldots$ ARE divisble by 3.

Soln: The key is to note that $n^2 - 1 = n^2 - 1^2 = (n+1)(n-1)$. If n is not a multiple of 3, then it is either 1 more than a multiple of 3 (such as 1,4,7,10,etc.,) or 1 less than a multiple of 3 (such as 2,5,8,11,etc.,). If n is like 1,4,7,etc., then $n - 1$ is divisible by 3 (such as 0,3,6,etc.,). If n is like 2,5,8,etc., then $n + 1$ is divisible by 3 (such as 3,6,9,etc.,). Thus $n^2 - 1 = (n+1)(n-1)$ will always have a multiple of 3 in it.