11/10/2010 Fall 10, College Algebra I Quiz 8 solutions Sitaraman
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1. Check if the following equation represents a function using vertical line test: $x^2 + y^2 = 1$.

Soln:

The graph of this is a circle with radius 1 and center (0,0). Each vertical line $x = k$ cuts the graph in two places (except for the two lines at the endpoints $x = 1$ and $x = -1$). So this equation does not represent a function.

Note that to show something is not a function it is enough to find just *one* vertical line which cuts the graph in two places. For example here the $y-$axis cuts the circle in two places.

It is not enough to solve the equation for y and say $y = \pm\sqrt{1 - x^2}$ and that each x gives two $y-$values, namely $\sqrt{1 - x^2}$ and $-\sqrt{1 - x^2}$. This is because the question explicitly asks to use vertical line test (but I gave partial credit if you did that).

2. Given that $f(x) = \frac{x}{x+1}$ find its domain.

Soln:

To find the domain we need to find the points where the function is possibly undefined and exclude those points.

The only values of x that might not be valid for this function are those that make the denominator equal 0.

Setting denominator $= x + 1 = 0$, we get $x = -1$.

So we exclude -1 from the domain and say that all real numbers except -1 are in domain.

3. (10 points) With $f(x)$ as in problem 2, $g(x) = x+2$, find the following: $(f + g)(x), (f + g)(0), (f/g)(x), f \circ g(x)$ $(i.e,\ f(g(x))), f(g(1))$.

Soln:

$(f+g)(x) = f(x)+g(x) = \frac{x}{x+1}+x+2.$ $\quad f+g(0) = \frac{0}{0+1}+(0+2) = 2.$

$(f/g)(x) = \frac{x}{x+1}/(x + 2) = \frac{x}{(x+1)(x+2)}.$

$f\circ g(x) = f(g(x)) = \frac{g(x)}{g(x)+1} = \frac{x+2}{(x+2)+1} = \frac{x+2}{x+3}$ $\quad f\circ g(1) = \frac{1+2}{1+3} = 3/4.$

4 (10 points). Graph the function $f(x) = 2x + 3$. Find its $x-$intercepts –i.e, values where $y = f(x) = 0$. Check whether the function is even or odd.

The graph is a straight line passing through (0,3) and (1,5).

The $x-$intercept is given by $2x + 3 = 0$ which gives $x = -3/2$.

It is neither even nor odd. When you replace x by $-x$ you don't get $f(x)$ or $f(-x)$.

$f(-x) = 2(-x)+3 = -2x+3$ and it is neither equal to $f(x) = 2x+3$ nor equal to $-f(x) = -2x - 3$.

5.(bonus 10 points) A wire of length x cm is bent in the shape of an equilateral triangle. Write the area of the triangle as a function of x.

See graph in file titled "Quiz 8 Graph" on update page.

The length of each side is $x/3$.

Area is base times height.

The height of the triangle is obtained by drawing a perpendicular line to the base through the vertex.

This divides it into two right equal triangles, each with base equal to half the base of the big triangle, i.e, $\frac{1}{2}(x/3) = x/6$ and hypotenuse equals the length of the big triangle, namely $x/3$.

Using Pythagoras theorem, we get the height of the triangle as $\sqrt{(x/3)^2 - (x/6)^2} = \sqrt{3x^2/36} = x/\sqrt{12}$.

So the area is

$A(x) = (1/2)(base)(height) = (1/2)(x/3)(x/\sqrt{12}) = \frac{x^2}{6\sqrt{12}}$.