10/27/2010 Fall 10, College Algebra I Quiz 7 Solutions Sitaraman
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1. Find the equation of line perpendicular to $y = 2x + 1$ and passing through (1,1).

Soln: Slope of given line is 2. Slope of perpendicular line will be $-1/2$. So equation of perpendicular line will be of the form $y = (-1/2)x + b$ where b is the $y-$intercept. Plugging in (1,1) we get $1 = (-1/2)(1) + b$ and so $b = 3/2$. So equation of perpendicular line passing through (1,1) will be $y = (-1/2)x + (3/2) = \frac{-x+3}{2}$ which can also be written as $2y = -x + 3$.

2. The cost of living in a city (C) is known to have a linear relationship with the price of the houses (p). If C is 1000 dollars per month when p is 100,000, and C is 1500 when p is 200,000, find the equation relating C and p.

Soln: Since they are known to satisfy a linear relationship, let $C = mp+b$. We have two points on the line given to us, namely$(100,000, 1000)$ and $(200,000, 1500)$. The slope $m = \frac{1500-1000}{200{,}000-100{,}000} = 1/200$. So $C = \frac{p}{200} + b$. Plugging in (100,000,1000) we get $1000 = (100,000/2000) + b$ which gives $b = 500$. So the equation is $C = \frac{p}{200} + 500$.

3. (10 points) Draw the graph of circle (approximate sketch is enough) representing the equation $x^2 + y^2 + 2y = 3$, marking the center, x and y intercepts as well as the length of the radius.

Soln: Collecting x and y terms we get $x^2+(y^2+2y) = 3$. Completing the square for y^2+2y we get $x^2+(y^2+2y+1^2) = 3+1^2 \Rightarrow x^2+(y+1)^2 = 2^2$. So we have $x - h = x$ and $y - k = y + 1$ and so the center is $(h, k) = (0, -1)$ and the radius is $r = \sqrt{2^2} = 2$. The intercepts are obtained by putting $x = 0$ and $y = 0$. We get the x intercepts by solving $x^2 + 0^2 + 0 = 3$ which gives $x = \pm\sqrt{3}$. We get $y-$intercepts by solving $0^2 + y^2 + 2y = 3$. This is same as $y^2 + 2y - 3 = 0$ which can be factored to get $(y + 3)(y - 1) = 0$ which gives $y = -3$ and $y = 1$.

4 (10 points). Write the equation that represents the following variation: Force varies inversely as the square of the distance d. If force is 100 Newtons when distance is 2, find the constant of variation and then find the force when distance is 4 units.

Soln: $F = \frac{k}{d^2}$ where k is the constant of variation. Plugging in $F = 100, d = 2$, we get $100 = k/2^2 = k/4$ which means $k = 400$. Now we can find F when $d = 4$ as follows: $F = k/4^2 = 400/16 = 25$ Newtons. Notice how when distance is doubled, force is reduced by a factor of 4.

5.(bonus 10 points) The diameter of a circle is the line segment with endpoints $(\sqrt{2}-\sqrt{3}, \sqrt{5}-\sqrt{7})$ and $(\sqrt{2}+\sqrt{3}, \sqrt{5}+\sqrt{7})$. Without graphing the points or using distance formula, show that the center of the circle is $(\sqrt{2}, \sqrt{5})$. After finding the center, use distance formula to find the radius.

The center is the midpoint of the diameter and it can be found using midpoint formula:

$$(h,k) = \left(\frac{(\sqrt{2}-\sqrt{3})+(\sqrt{2}+\sqrt{3})}{2}, \frac{(\sqrt{5}-\sqrt{7})+(\sqrt{5}+\sqrt{7})}{2} \right)$$

$$= (\sqrt{2}, \sqrt{5}).$$

The radius can be found by taking the distance from the center to either one of the endpoints of the diameter. Let us take the distance from $(\sqrt{2}, \sqrt{5})$ to $(\sqrt{2}-\sqrt{3}, \sqrt{5}-\sqrt{7})$. You get

$$r = \sqrt{(\sqrt{2}-(\sqrt{2}-\sqrt{3}))^2 + (\sqrt{5}-(\sqrt{5}-\sqrt{7}))^2} = \sqrt{(\sqrt{3})^2 + (\sqrt{7})^2}$$

$$= \sqrt{10}$$