10/20/2010 Fall 10, College Algebra I Quiz 6 Solutions Sitaraman
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1. (10 points) Find the midpoint of the two points (-1,3) and (4,-2) using the midpoint formula. Find the distance between them using the distance formula.

Soln: Using midpoint formula the midpoint is

$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{-1+4}{2},\frac{3+-2}{2}\right)=(3/2,1/2)=(1.5,0.5)$$

Using distance formula the distance between the points is

$$\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}=\sqrt{(-1-4)^2+(3--2)^2}=\sqrt{(-5)^2+5^2}$$

$$=\sqrt{50}=7.071$$

2. (10 points) Draw the graph (approximate sketch is enough) representing the equation $x^2-y=4$, marking at least 3 points on it. Find the x and y intercepts. Check for symmetry about the x, y axes and the origin (0,0).

Soln: The graph has the shape of a bowl with its bottom point at (0,-4) which is also the $y-$intercept (Put $x=0$ in the equation). The $x-$ intercept is given by putting $y=0$ which gives $x^2=4\Rightarrow x=\pm 2$. So we get two more points on the graph (-2,0) and (2,0) which are also the $x-$ intercepts. Checking for symmetry about the x axis by changing the sign of y we get $x^2-(-y)=4$ which is same as $x^2+y=4$. Thus the equation is changed when switching the signs of y and so the curve is not symmetrical about the $x-$axis. Doing the same for the y axis by switching the signs of x we get $(-x)^2-y=4$ which gives $x^2-y=4$ and so the equation is unchanged. Thus the curve is indeed symmetric about the $y-$axis. It is not symmetrical about the origin (0,0) because when you change signs of x and y at same time the equation becomes $x^2+y=4$.

3. (10 points) Write the equation of the line with slope -1 and $y-$ intercept 3.5. Find its $x-$intercept.

Soln: Put $m=-1$ and $b=3.5$ in $y=mx+b$ to get $y=-x+3.5$. Its x intercept is given by putting $y=0$. We get $0=-x+3.5$ which means $x=3.5$. So 3.5 is also the $x-$intercept.

4.(bonus 10 points) A corvette and a porsche leave a place at the same time. The corvette goes at an average speed of 80 mph along the direction given by the line $y=2x$. The Porsche goes at an average speed of 60 mph along $y=x$. Write down the co-ordinates (in terms of t) for each car after t hours, with the co-ordinates of the starting place being (0,0). Find an expression (in terms of t) for the distance between them after t hours.

Let t be the time in hours. Then the corvette goes $80t$ miles (use distance = speed times time) along $y=2x$. The distance from (0,0) travelled by a point on $y=2x$ is given by using the distance formula between the points (0,0) and $(x,2x)$. This is given by $\sqrt{x^2+(2x)^2}=\sqrt{5x^2}=\sqrt{5}x$. So we get that $\sqrt{5}x=80t$ and so $x=\frac{80}{\sqrt{5}}t$. Since it is on $y=2x$ its $y-$co-ordinate is $\frac{2(80t)}{\sqrt{5}}=\frac{160t}{\sqrt{5}}$ and its co-ordinate will be $\left(\frac{80t}{\sqrt{5}},\frac{160t}{\sqrt{5}}\right)$). The Porsche goes a distance of $60t$ miles along $y=x$. The distance from (0,0) travelled by a point on $y=x$ is given by using the distance formula between the points (0,0) and (x,x). This is given by $\sqrt{x^2+(x)^2}=\sqrt{2x^2}=\sqrt{2}x$. So we get that $\sqrt{2}x=60t$ and so $x=\frac{60}{\sqrt{2}}t$. Since it is on $y=x$ its $y-$co-ordinate is $\frac{(60t)}{\sqrt{2}}$ as well and its co-ordinate will be $\left(\frac{60t}{\sqrt{2}},\frac{60t}{\sqrt{2}}\right)$. The distance between them using distance formula will be

$$\sqrt{\left(\frac{60t}{\sqrt{2}}-\frac{80t}{\sqrt{5}}\right)^2,\left(\frac{60t}{\sqrt{2}}-\frac{160t}{\sqrt{5}}\right)^2}$$