9/3/2010 Fall 10, College Algebra I Quiz 2 Solutions Sitaraman
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1.(10 points) Divide $x^4 - 1$ by $x - 1$ using (a) long division and (b) synthetic division. [Bonus 5 points: Is $x-1$ a factor of $x^4+x^3+x^2+x+1$?]

Soln: (a) Long division: [Note that I have put the quotient on the right instead of the top]. You get

$$
\begin{array}{llll}
x-1 & [& x^4+0x^3+0x^2+0x+1 &] \quad x^3+x^2+x+1 \\
 & & x^4-x^3 & \\
 & & \quad x^3+0x^2 & \\
 & & \quad x^3-x^2 & \\
 & & \qquad x^2+0x & \\
 & & \qquad x^2-x & \\
 & & \qquad\quad x-1 & \\
 & & \qquad\quad x-1 & \\
 & & \qquad\qquad 0 &
\end{array}
$$

(b) Synthetic division:

$$
\begin{array}{c|ccccc}
1 & 1 & 0 & 0 & 0 & -1 \\
 & 0 & 1 & 1 & 1 & 1 \\
 & 1 & 1 & 1 & 1 & 0
\end{array}
$$

The quotient is found from the bottom row, and you start with x^3 [when you divide x^4 by x you get x^3]. You write the rest in descending order to get $x^3 + x^2 + x + 1$ as quotient.

Since the remainder is 0, $x - 1$ is a factor of $x^4 - 1$. In other words, $x - 1$ exactly divides it.

2.Simplify $(3x + 20)^2$ usign the formula $(A + B)^2 = A^2 + B^2 + 2AB$.

Soln:YOU MUST USE THE FORMULA!! JUST MULTIPLYING TERM BY TERM IS NOT ENOUGH. Put $3x = A$ and $20 = B$. Then the answer is $(3x + 20)^2 = (3x)^2 + 20^2 + 2(3x)(20) = 9x^2 + 120x + 400$.

3. Factor $27y^3 - 64$ by writing it as a difference of cubes and using the formula for factoring $A^3 - B^3$.

Soln: $27y^3 - 64 = (3y)^3 - 4^3$. Set $A = 3y$ and $B = 4$ in $A^3 - B^3 = (A-B)(A^2 + AB + B^2)$. You get the answer as $(3y-4)((3y)^2 + (3y)4 + 4^2) = (3y-4)(9y^2 + 12y + 16)$.

4. Factor $4x^2 + 20x + 25$ by expressing it as a square [Compare this expression to $A^2 + 2AB + B^2$ which is a square].

Soln:Setting $A = 2x$ and $B = 5$ we get $4x^2 + 20x + 25 = (2x)^2 + 2(2x)5 + 5^2 = (2x+5)^2$.

5.Factor $x^2 + 502x + 1000$.

Soln: Looking at the coefficients 1000 and 502 we see that $1000 = 500 \times 2$ and $502 = 500 + 2$. So $x^2 + 502x + 1000 = (x+2)(x+500)$.