11/24/2010 Fall 10, College Algebra I Quiz 10 Solutions Sitaraman
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1. (16 points) Graph the exponential function $y = 4^x$, labelling the x and y intercepts (if any). When is it positive and when is it negative? Graph $y = \frac{1}{4^x}$ by modifying the graph of $y = 4^x$. Find out value of x for which the function $f(x) = 4^x$ equals 256.

Soln:

The graph of 4^x crosses the $y-$axis at $4^0 = 1$ but it never touches the $x-$axis. This is because for all x the value of 4^x is a positive number. [When x is a big negative number like -100 you get $4^{-100} = \frac{1}{4^{100}}$ which is a very small number but nevertheless not zero. Same is true for all negative numbers]. On the right of the $y-$axis it increases very rapidly.

The graph is always above the $x-$axis. So the function is always positive (i.e, positive for all real number values of x) and never negative.

The graph of $y = 4^{-x}$ can be obtained by reflecting the graph of 4^x about the $y-$axis.

Since 256 is same as 4^4, from $4^x = 4^4$ we get $x = 4$.

2. (14 points) A certain mutual funds gives an annual return of 5 percent. How much money would you have in 10 years if you invested 1000 dollars now? How much should you invest now to have 20000 dollars after 10 years?

Soln:

5 percent is same as 0.05. So $r = 0.05$ in compound interest formula.
The Amount after t years is given by $A(t) = P(1+0.05)^t = P(1.05)^t$.
If $P = 1000$, when $t = 10$ we get $A(10) = 1000(1.05)^{10}$
$= 1000(1.62889) = 1628.89$ dollars.
If we want $A(10) = 20,000$, then we need $20,000 = P(1.05)^{10}$ so
$P = 20,000/(1.05)^{10} = 20,000/1.62889 = 12,278.30$ dollars.

3.(bonus 10 points) Show that regardless of the rate of interest r the time it takes for the initial amount P to double (that is, $A(t) = 2P$) is the same as the time it takes it takes to go from $2P$ to $4P$ and so on. For example, in problem 2, show that after 14.2067 years, 1000 dollars becomes 2000 dollars and after another 14.2067 years, 2000 dollars becomes 4000 dollars. Then show that the same should be true for any amount P and any rate of interest r.

The basic idea is that to compute compound interest we multiply P by $1 + r$ each time interest is compounded.

If it is compounded annually, then after t years you would have multiplied P by $(1+r)(1+r)...(1+r)$ [$1+r$ multiplied by itself t times] or by $(1+r)^t$ to get the amount $A(t)$ as $P(1+r)^t$.

So if, after a certain time T the multiple $(1+r)(1+r)...(1+r) = (1+r)^T$ equals 2, then when you multiply this by P you get $2P$. But then if you start with $2P$ and multiply T times by $(1+r)$ you should get $2P$ times 2 or $4P$.

Here is the solution in mathematical language:

It is easy to show that $1000(1.05)^{14.2067} = 2000$.

Also easy to show that $2000(1.05)^{14.2067} = 4000$.

But this was just for example. let us look at general situation.

Now let T be the amount it takes for P to become $2P$.

So $A(T) = P(1+r)^T = 2P$.

After cancelling P on both sides we get $(1+r)^T = 2$.

Now, if you start with $2P$, the amount after another T years equals $A(T) = (2P)(1+r)^T$ because now the initial amount is $2P$ and you simply replace P by $2P$ in the formula.

But $(1+r)^T = 2$, so we get $A(T) = (2P)(2) = 4P$.