Fall 09, Precalculus (Honors) Quiz 9 Howard University Mathematics
Math 007-9 Sitaraman 11/13/09

SOLUTIONS

1. Given that in a right angled triangle $a = 5, b = 12$ find A, B.

Soln: We get $c = \sqrt{5^2 + 12^2} = \sqrt{169} = 13$. So $A = sin^{-1}\frac{5}{13} = 22.62$ degrees. $B = 90 - A = 67.38$ degrees.

2.If a 6 foot tall person casts a shadow of exactly 6 feet at a certain time of the day, find the angle of elevation of the sun at that time.

Soln:If θ is the angle of elevation, we get $tan\theta = 6/6 = 1$. Thus $\theta = tan^{-1}1 = 45$ degrees.

3. Calculate $sin(2cos^{-1}(0.6))$.

Soln: Let $\alpha = sin^{-1}0.6$. Then $sin\alpha = 0.6$ and $cos\alpha = \sqrt{1 - sin^2\alpha} = \sqrt{1 - (0.6)^2} = 0.8$. (We take the positive square root because α being equal to sin^{-1} of an angle is between $-\pi/2$ and $\pi/2$ and cosine is positive in that region]. Then $sin(2\alpha) = 2sin\alpha cos\alpha = 2(0.6)(0.8) = 0.96$.

4. Write $cos(3A)sin(4A)$ as a sum or difference.

Soln: Using the formula $sin(\alpha+\beta) + sin(\alpha-\beta) = 2sin\alpha cos\beta$, with $\alpha = 4A$ and $\beta = 3A$, we get that $2sin4Acos3A = sin(3A+4A)+sin(4A-3A) = sin7A + sinA$. Therefore $cos3Asin4A = \frac{1}{2}(sinA + sin7A)$.

5.Find all θ in $[0, 2\pi)$ such that $sin(3A) = 1$.

Soln: Since $0 \le \theta < 2\pi$ we get $0 \le 3\theta < 6\pi$. In this range (i.e, from 0 to $6\pi = 1080$), we get (looking at the graph) $sin3\theta = 1$ for $3\theta = \pi/2, 5\pi/2, 9\pi/2$ or $90, 450$ and 810 degrees. So by dividing all by 3, we get that $\theta = \pi/6, 5\pi/6, 9\pi/6$ or $30, 150, 270$ as the solutions. Note that all three are within $[0, 2\pi)$.

6.Find all values of θ such that $cos\theta = 1$.

Soln:First we find the values in $[0, 2\pi)$ for which this is true. From graph of cosine function, we can see that this happens for 0 only. Now we get all other values by adding multiples of 2π, getting $\theta = 0+n(2\pi) = 2n\pi$ where $n = \ldots - 3, -2, -1, 0, 1, 2, 3, \ldots$