Fall 09, Precalculus (Honors) Quiz 8 Howard University Mathematics
Math 007-9 Sitaraman 11/6/09

SOLUTIONS

1. Show that $2sin75cos15 = sin90 + sin60 = 1.866$ without using the calculator.

Soln: This requires a product to sum formula. We know $sin(A + B) + sin(A - B) = (sinAcosB + cosAsinB) + (sinAcosB - cosAsinB)$ and this reduces to $2sinAcosB$. So writing $sin90 = sin(75 + 15)$ and $sin60 = sin(75 - 15)$ (i.e, A = 75, B = 15) we get $2sin75cos15 = sin90 + sin60 = 2.732$.

2.Find the value of $cos105 + cos15$ (without calculator) by writing it as a product.

Soln:We know $cos(A + B) + cos(A - B) = 2cosAcosB$. So we need to find out what A and B are, given that $A + B = 105$ and $A - B = 15$. Solving, we get $A = 60, B = 45$. So $cos105 + cos15 = 2cos60cos45 = 2(\frac{1}{2})(\frac{\sqrt{2}}{2}) = \sqrt{2}/2$.

3. Calculate $cos(2sin^{-1}\frac{3}{5})$ without using calculator.

Soln: Let $\alpha = sin^{-1}(\frac{3}{5})$. Then $sin\alpha = \frac{3}{5}$. Now $cos(2\alpha) = 1 - 2sin^2\alpha = 1 - 2(3/5)^2 = 1 - 2(9/25) = 7/25$.

4. Find θ such that $cos(2\theta) = 0$ and $0 \leq \theta < 2\pi$.

Soln: First find the values in $[0, 2\pi)$ for which cosine is zero and then get the others by adding multiples of 2π. The values (looking at the graph) for which cosine is zero inside $[0, 2\pi)$ are 90 and 270. i.e, $\pi/2$ and $3\pi/2$. Adding multiples of 360 (i.e, 2π radians) to these, we get 450 and 630 ($5\pi/2, 7\pi/2$.) We stop with 630 because our values should be within 720. So $2\theta = 90, 270, 450, 630$ and $\theta = 45, 135, 225, 315$ or $\pi/4, 3\pi/4, 5\pi/4, 7\pi/4$.

5. Find all possible values of θ such that $tan\theta = \sqrt{3}$.

Soln: $tan\frac{\pi}{3} = \sqrt{3}$. Since $tan\theta$ is periodic with period π, we get $\theta = \frac{\pi}{3} + n\pi$ where $n = ... - 3, -2, -1, 0, 1, 2, 3, ...$

6. Find θ in $[0, 2\pi)$ such that $cos(2\theta) + cos(\theta) = 0$.

Soln:

Find θ in $[0, 2\pi)$ such that $cos(2\theta) + cos(\theta) = 0$.

First we convert the sum $cos(2\theta) + cos(\theta)$ to a product using the formula $cos(A+B) + cos(A-B) = 2cosAcosB$. Letting $A+B = 2\theta$ and $A - B = \theta$ we get $A = \frac{3\theta}{2}$ and $B = \frac{\theta}{2}$. So we get $cos(2\theta) + cos(\theta) = 2cos(\frac{3\theta}{2})cos(\frac{\theta}{2}) = 0$ which means $cos(\frac{3\theta}{2}) = 0$ or $cos(\frac{\theta}{2}) = 0$. First we look at $cos(\frac{3\theta}{2}) = 0$. Now θ in $[0, 2\pi)$ means $3\theta/2$ is in $[0, 3\pi]$ (Multiply both sides by $3/2$]. In $[0, 3\pi]$ we have $3\theta/2 = \pi/2, 3\pi/2, 5\pi/2$ for which cosine is 0. This means for $\theta = \pi/3, \pi, 5\pi/3$ in $[0, 2\pi)$ are in the solution.

Now we look at $cos(\frac{\theta}{2}) = 0$. Using the same argument as above, we get the values of $\theta/2 = \pi/2$ as the only solution in $[0, \pi)$ and so $\theta = \pi$ is the only solution in $[0, 2\pi]$.

Putting them together we get the list of solutions as $\pi/3, 5\pi/3, \pi$.