Fall 09, Precalculus (Honors) Test 1 Howard University Mathematics
Math 007-9 Sitaraman 9/23/09

SOLUTIONS

1. Given that $f(x) = x^2 + 1$, and $g(x) = \sqrt{2x}$ find $f \circ g(x)$ and then evaluate $f \circ g(2)$. What can you say about $f \circ g(-1)$?

Soln: $f \circ g(x) = f(g(x)) = g(x)^2 + 1 = (\sqrt{2x})^2 + 1 = 2x + 1$. Now $f \circ g(2) = 2(2) + 1 = 5$. As for $f \circ g(-1)$, it is not defined because -1 is not in the domain of g. Remember that the domain of $f \circ g$ consists of elements x in the domain of g for which $g(x)$ is in the domain of f.

2. Given that $f(x) = \frac{x+5}{x-5}$ find f^{-1} and also find the domain of f and f^{-1}.

Soln: Start by switching x and y in $f(x)$ getting $x = \frac{y+5}{y-5}$. Now solve for y : .From $x = \frac{y+5}{y-5}$ you get $x(y-5) = y + 5$. Gathering terms with y in it, you get $xy - y = 5 + 5x$ from which you get $y = \frac{5(x+1)}{x-1}$. So $f^{-1}(x) = \frac{5(x+1)}{x-1}$. The domain of f is all real x other than 5. The domain of f^{-1} is all x other than 1.

3. Write as a single logarithm: $lnx + 2ln(x-1) - ln(3x+2)$.

Soln: Using power rule we get $2ln(x-1) = ln(x-1)^2$. Plugging this into the expression $lnx + 2ln(x-1) - ln(3x+2) = lnx + ln(x-1)^2 - ln(3x+2) = ln\left[\frac{x(x-1)^2}{3x+2}\right]$.

4. Solve the logarithmic equation $log_x 27 = 3$.

Soln: This can be solved in two ways. A. You can directly convert this to an exponential equation using the definition of logarithms and solve it. You get $x^3 = 27$ which leads to $x = 3$ by taking cube root of both sides. Or else you can use change of base formula to get $log27/logx = 3$ which results in $logx = log27/3 = (1/3)log27 = log27^{1/3} = log3$. Thus $logx = log3$ which means $x = 3$.

5. Solve the following exponential equation: $7^{2x-1} = 49$.

Soln: Since $49 = 7^2$, we get $7^{2x-1} = 7^2$ which gives $2x - 1 = 2$ from which we solve for x to get $x = 1.5$.

6.(20 points) Find how long it would take for money to triple if it is invested in a stock fund that gives a 10 percent annual return.

Soln: $A(t) = 3P$, $r = 0.1$, $n = 1$, $t =?$. We get the equation $3P = P(1+0.1)^t = P(1.1)^t$. Cancelling P on both sides, we get $3 = 1.1^t$ which means $ln3 = t(ln1.1)$ taking logarithms of both sides. Solving, we get $t = ln3/ln1.1 = 11.5267$ or about 11 years and 6 months and about 1 day.

7. (20 points) Given that a pizza pie is cooling according to the equation $T(t) = 80 + 220e^{-0.05t}$ where t is in minutes, find the initial temperature. How long would it take to reach 100 degrees? [Assume the unit is Fahrenheit].

Soln: To find the initial temperature put $t = 0$ in the equation $T(t) = 80 + 220e^{-0.05t}$ to get $T(0) = 80 + 220e^{-0.05(0)} = 80 + 220 = 300$ degrees F. To find when it would reach 100 degrees, put $T(t) = 100$ in the same equation to get $100 = 80 + 220e^{-0.05t}$. Solving, we get $20 = 220e^{-0.05t} \Rightarrow ln(.091) = -0.05t$ which gives $t = 47.9379$ or 47 minutes and about 56 seconds.

8. Carbon-14 decays at the rate of 0.000124 or 0.0124 percent. If a tree contains 10 grams of carbon-14 now, how much would it contain after 5600 years?

Soln: The equation of decay is $A(t) = A_0e^{-0.000124t}$. Putting $A_0 = 10$, we get $A(5600) = 10e^{-0.000124\times5600} = 5$ grams. [Note that this is half of original amount! Half-life of carbon-14 is 5600 years. You can use amount of carbon-14 present to find the age of ancient objects].

9. [Challenge problem, 20 points extra credit] A King decides to punish a particularly greedy minister using money. He tells the minister that he could take gold coins fromn the treasury every day, starting with one gold coin on the first day. The condition is that each day he should take double of the previous day and he should carry all the coins to his home on his own back. If each coin weighs an ounce, and 16 ounces is a pound, how many days would it take before the weight exceeds a tonne (2000 lbs)? You must use logarithms to find the answer.

Soln: After n days, the weight is $(1/16)2^{n-1}$ pounds because the first day it is 1 $(= 2^{1-1})$ coin, second day it is 2 $(= 2^{2-1})$ coins, 3rd day it is

4 ($= 2^{3-1}$) coins, and so on. We have to divide by 16 because each coin is one-sixteenth of a pound. So we want $\frac{2^{n-1}}{16} > 2000 \Rightarrow 2^{n-1} > 32000$ which means $n - 1 > ln(32000)/ln2 = 14.97 \Rightarrow n > 15.97$. So on the 16th day the weight would exceed a tonne.