Fall 09, Test 2, Probability and Statistics I Math 189-1 Sitaraman
10/16/09 Howard University Mathematics

SOLUTIONS

1. A coin is tossed, a card is picked from a deck of 52, and a die is rolled at the same time. Find the probability of getting a head on the coin, an ace and a six on the die.

Soln: These are all independent events. The combined probability can be found by multiplying the individual probabilities together. We get :(1/2)(1/13)(1/6) = 1/156 = 0.00641.

2. (20 points) A shipment of two boxes, each containing six telephones, is received by a store. Box 1 contains one defective phone and box 2 contains 2 defective phones. After the boxes are unpacked, a phone is selected and found to be defective. Find the probability that it came from box 2.

Soln: Let B_1 represent the box 1 and B_2 represent box 2. Let D represent the defective phone and ND represent the normal phone. Using Bayes' theorem,

$$\begin{aligned} P(B_2|D) &= \frac{P(B_2)P(D|B_2)}{P(B_1)P(D|B_1) + P(B_2)P(D|B_2)} \\ &= \frac{\frac{1}{2} \times \frac{2}{6}}{\frac{1}{2} \times \frac{1}{6} + \frac{1}{2} \times \frac{2}{6}} \\ &= (1/6)/(3/12) = 2/3. \end{aligned}$$

3. On a game show, a contestant can select one of four boxes. Box 1 contains one 100 dollar bill and nine 1 dollar bills. Box 2 contains two 100 dollar bill and eight 1 dollar bills. Box 3 contains three 100 dollar bill and seven 1 dollar bills. Box 4 contains five 100 dollar bill and five 1 dollar bills. Find the probability of drawing a 100 dollar bill if the contestant selects a box at random and then a bill from that box at random.

Answer: $P(100) = P(100 \cap B_1) + P(100 \cap B_2) + P(100 \cap B_3) + P(100 \cap B_4)$. Using conditional probability on each one of these intersections, we get $P(100) = P(B_1)P(100|B_1) + P(B_2)P(100|B_2) + P(B_3)P(100|B_3) + P(B_4)P(100|B_4)$. Noting that the probability of selecting any one of the boxes is 1/4, and the probabilities of selecting a 100 dollar bill from the four boxes are $1/10 = 0.1$, $2/10 = 0.2$, $3/10 = 0.3$ and $5/10 = 0.5$ respectively, we get $P(100) = (1/4)(0.1+0.2+0.3+0.5) = 1.1/4 = 0.275$.

4. Check whether the $f(x)$ represents a probability mass function for the random variable X:

X	0	2	4	6
$f(x)$	-1.0	1.5	0.3	0.2

Soln:No, it is not a p.m.f because even though the probabilities add up to 1, probability cannot be 1.5 (> 1) or -1.0 (< 0).

5. If a player rolls two dice and gets a sum of 2 or 12, she gets \$20. If she gets a 7, she wins \$5. The cost to play the game is \$3. What is the expected value of the game, i.e, the money that she is expected to win minus the price to play?

Soln: 2 comes up only if both dice turn up 1 and so $P(2) = 1/36$. 12 can be obtained in only one way–6 and 6. So $P(12) = 1/36$. 7 can occur by getting 1 and 6 on the dice, or 2 and 5 or 3 and 4 or 4 and 3 or 5 and 2 or 6 and 1. Thus $P(7) = 1/6$. Now the values of the random variable are 20 for 2 and 12 and 5 for 7. So $E(X) = 20(\frac{1}{36} + \frac{1}{36}) + 5(1/6) = 70/36 = 1.94$ dollars. So the player can expect to win only 1.94 dollars which means she would lose 1.06 dollars.

6.(20 points) Find the mean, variance and standard deviation of the random variable X which counts the number of tails in a toss of three coins.

Soln: $X = 0, 1, 2, 3$. The pmf is given by $f(0) = 1/2^3 = 1/8, f(1) = 3/8, f(2) = 3/8, f(3) = 1/8$. Mean is $0 \times \frac{1}{8} + 1 \times \frac{3}{8} + 2 \times \frac{3}{8} + 3 \times \frac{1}{8} = 12/8 = 1.5$. The variance $\sigma^2 = E((X-\mu)^2) = (0-1.5)^2 \times \frac{1}{8} + (1-1.5)^2 \times \frac{3}{8} + (2-1.5)^2 \times \frac{3}{8} + (3-1.5)^2 \times \frac{1}{8} = 6/8 = 0.75$. So variance $= \sigma^2 = 0.75$ and the standard deviation $\sigma = \sqrt{0.75} = 0.866$.

7. Given that $E(X+1) = 5$ and $E(X^2) = 18$ find the mean μ and variance σ^2 of X.

Soln: $E(X+1) = E(X) + 1$ by the properites of $E(X)$. Therefore $\mu = E(X) = 5 - 1 = 4$. Now $\sigma^2 = E(X^2) - \mu^2 = 18 - 4^2 = 2$.

8. If a student randomly guesses at five multiple choice questions, find the probability that the student gets exactly 3 correct. Each question has five possible choices out of which only one is correct.

Soln:This is given by the binomial distribution B(5,0.2) because there are five experiments and probability of success in each is $1/5 = 0.2$. Then $P(3) = 5C_3(0.2)^3(0.8)^2 = 10(0.008)(0.64) = 0.0512$. So the chances of getting exactly 3 correct answers is 5.12 percent.

9. (Challenge problem –20 points extra credit) Prove the following statements for 3 sets A, B, C : 1. $P(A \cap B \cap C) = P(A|B \cap C)P(B|C)P(C)$ 2. $P(A|AUB) = \frac{P(A)}{P(A)+P(B)}$ if A, B are mutually exclusive.

Soln: We have $P(A \cap B \cap C) = P(A|B \cap C)P(B \cap C)$ by definition of conditional probability. Similarly $P(B \cap C) = P(B|C)P(C)$. Putting the two together we get the statement 1. For statement 2, use defitnion of conditional probability and the fact that $A \cap (AUB) = A$ and also that $P(AUB) = P(A) + P(B)$ for two mutually exclusive events.