Fall 09, Test 1, Probability and Statistics I Math 189-1 Sitaraman
9/23/09 Howard University Mathematics

SOLUTIONS

1. Let $p(q)$ be the probability that a positive integer q is a prime. Let $f(n)$ be the relative frequency of primes, found by $f(n) = \frac{prime(n)}{n}$ where $prime(n) =$ the number of primes upto and including n. Find the relative frequency for each n upto $n = 20$ and compare with the well-known estimate $p(n) = 1/logn$. [Hint: Start by making a list of primes upto 20. Prime numbers are those that are divisible only by 1. The first few are 2,3,5,7,...].

The primes upto 20 are 2,3,5,7,11,13,17,19. The relative frequency is given in this table (for n upto 12):

n	1	2	3	4	5	6	7	8	9	10
$f(n)$	0	1/2	2/3	2/4	3/5	3/6	4/7	4/8	4/9	4/10

The value of $f(n)$ as $n \to \infty$ behaves like $1/logn$. So we could say the probability $p(q)$ is about $1/logq$ as $q \to \infty$.

2. Show that the probability of getting at least one head in a toss of 8 coins is $1 - (1/2)^8 = 1 - 2^{-8} = .996$.

Let H represent the event that there is atleast one head. Then $HUT = S$ where T is the event that all 8 are tails. Since these are mutually exclusive, $P(HUT) = P(S) = P(H) + P(T)$. So $P(H) = P(S) - P(T) = 1 - (1/2)^8$ because the probability of all tails is just one event out of 2^8.

3. How many different sets of initials can be formed if each person has one first name, one middle name, and one last name? For instance, John Edward Johnson is JEJ.

Answer: 26^3. This is just number of ways to arrange 26 letters in 3 places with replacement.

4. (20 points) Out of 12 computer modems shipped by a company, 3 are defective. If you randomly select 5 of the 12 modems, what is the probability that exactly 2 of them are defective?

Number of ways to choose 2 defective and 3 good modems is $3C_2 = 3$ and $9C_3 = (9\times8\times7)/6 = 84$ respectively. Once the 5 modems are chosen they can be arranged in $5! = 120$ ways. Total number of outcomes is $12P_5 = 12 \times 11 \times 10 \times 9 \times 8 = 95040$ So probability that exactly two are defective is $\frac{120 \times 84 \times 3}{95040} = 0.318$. Note that you would have gotten the same answer if you had looked only at number of combinations, and not considered permutations, because the 5! would have gotten cancelled, being common to both the numerator and denominator.

5.(20 points) Find the probability that a committee of {President, Vice President, Treasurer } chosen from a group of 10 people, let us say $P_1, P_2, \ldots, P_{10}$, would consist of P_1, P_2, and P_6.

Soln: Total number of outcomes is number of ways to arrange 10 people in 3 places which is $10P_3 = 720$. The 3 given people can be arranged in $3! = 6$ ways, so there can be six ways to choose a committee with these 3 people. So probability is $6/720 = 1/120 = 0.008333$. Again, you would have gotten the same answer had you looked only at the combinations, and not considered permutations.

6.Find the probability that you will win the lottery if you bought a hundred tickets and the lottery tickets have six different numbers from amongst $1, 2, 3, \ldots, 9$. In this lottery order in which numbers appear is relevant, and every possible combination appears in some ticket.

Soln: Total number of possible tickets is $9P_6 = 60480$. Number of favorable outcomes is 100. So probability of winning is $100/60480 = 0.00165$.

7. Find the probability that, given that the lakers beat the celtics in six games, they won the first 3 games.

Soln: Total number of possibilities, given that lakers won sixth game (and series) is just the number of ways to have lakers win 3 games and celtics win 2 games in the first five: LLCLC, LCLLC, etc., This is given by the number of distinguishable permutations of 5 things of which 2 are of one kind and 3 another, namely $\frac{5!}{3!2!} = 10$ ways. If lakers win first 3, next two can be only CC, so probability is just $1/10 = 0.1$

8. If $P(A) = 0.5, P(B) = 0.3$ and $P(A/B) = 0.4$ show that $P(AUB) = 0.68$

Soln:This is just $P(A) + P(B) - P(A \cap B) = 0.5 + 0.3 - 0.12 = 0.68$ because $P(A \cap B) = P(A/B)P(B)$.

9. (Challenge problem –20 points extra credit) prove Pascal's recursion formula: $nC_r = (n-1)C_r + (n-1)C_{r-1}$.

Soln: Call one of the n things as A. Number of ways to choose r things out of n can be split into two: Number of ways in which you can choose r things with A being one of them plus number of ways to choose r things with A not being one of them. The first part is simply the number of ways to choose $r-1$ things from the remaining $n-1$ things, or $(n-1)C_{r-1}$. The second part is the number of ways to choose r things out of the $n-1$ things (after excluding A) or $(n-1)C_r$.