Fall 09, Quiz 5, Probability and Statistics I Math 189-1 Sitaraman
11/13/09 Howard University Mathematics

SOLUTIONS

1.Assume that buses passing through an intersection every minute follow a Poisson distribution with mean 5. Find the probability that one will have to wait more than 30 seconds (0.5 minutes) for the first bus.

Soln: The distribution time for the random variable $W$ that gives the waiting time is an exponential function with $\theta$ being the parameter. In this case $\theta = 1/5 = 0.2$, the average waiting time. The probability of waiting more than 30 seconds or 0.5 minutes is
$P(W > 0.5) = 1 - P(W \leq 0.5) = 1 - F(0.5) = 1 - \int_0^{0.5} \frac{e^{-\frac{x}{0.2}}}{0.2} dx = e^{-\frac{0.5}{0.2}} = e^{-2.5} = 0.082$ or an 8.2 percent chance.

2.Show that the mean of the exponential distribution with parameter $\theta$ is equal to $\theta$ without using the moment generating function $M(t)$. i.e, use integration and the basic definition of mean.

Soln: By definition, $\mu = \int_0^\infty xf(x)dx = \int_0^\infty x \frac{e^{-\frac{x}{\theta}}}{\theta} dx$. This can be easily found to be equal to $\theta$ using integration by parts.

3. If $X$ measures the distance frrom the center of darts falling on a circular target of radius one foot and say (just for the sake of argument) that $X$ has a uniform distribution $U(0, 1)$, find the probability that a dart falls between 0 and 0.25 feet from center.

Soln: The distribution function for $U(0, 1)$ is given by $F(x) = \frac{x-0}{1-0} = x$. So $P(X \leq 0.25) = F(0.25) = 0.25$.

4. Suppose that the waiting time between calls at a booth follows a Gamma distribution with $\theta = 3$. Find the probability of waiting less than 5 minutes for the first two calls.

Soln: $\alpha = 2$. The pdf is $f(x) = \frac{x^{2-1} e^{-x/3}}{\Gamma(2) 3^2} = \frac{1}{9} x e^{-x/3}$. So we get that $P(X \leq 5) = F(5) = \int_0^5 \frac{1}{9} x e^{-x/3} dx = 1 - \frac{8}{3} e^{-5/3} = 0.4963$ or 49.63 percent.

5. Write down (you don't need to evaluate) the integral equation that will help you to find the 25th percentile (or quartile) of a random variable that has a gamma distribution with $\alpha = a$ and $\theta = b$. [Note: The

quartile is the value $q$ of $X$ such that probability of $X$ being less than $q$ is $1/4 = 0.25$.]

Soln: By definition, the quartile $q$ is given by the equation $P(X \leq q) = F(q) = 0.25$. In terms of the pdf, this is given by $\int_0^q \frac{x^{a-1}e^{-x/b}}{\Gamma(a)b^a} dx = 0.25$.

6. If $X$ is $\chi^2(15)$, find $P(X > 5.229)$.

Soln: From the tables, $P(X \leq 5.229) = 0.01$. So $P(X > 5.229) = 1 - 0.01 = 0.99$.