Fall 09, Quiz 3, Probability and Statistics I Math 189-1 Sitaraman
10/30/09 Howard University Mathematics

SOLUTIONS

1.Given that $M(t) = 0.3e^t + 0.4e^{2t} + 0.2e^{3t} + 0.1e^{4t}$ is the moment generating function of the random variable X, find μ, σ^2 and $P(1 \leq X \leq 2)$. Start by writing down the range of X.

Soln: $X = 1, 2, 3, 4$. The p.m.f is given by $f(1) = 0.3, f(2) = 0.4, f(3) = 0.2, f(4) = 0.1$. The mean is $\mu = \Sigma x f(x) = 1(0.3) + 2(0.4) + 3(0.2) + 4(0.1) = 0.3 + 0.8 + 0.6 + 0.4 = 2.1$. $\sigma^2 = \Sigma(x - \mu)^2 f(x) = E(X^2) - \mu^2$ This is $= 1(0.3) + 4(0.4) + 9(0.2) + 16(0.1) - (2.1)^2 = 0.3 + 1.6 + 1.8 + 1.6 - 4.41 = 5.3 - 4.41 = 0.89$. Also in this case $P(1 \leq X \leq 2) = P(1) + P(2) = 0.7$.

2. Given that X has a Poisson distribution with a mean of 2. Find $P(X = 3)$.

Soln: This is given by $P(X \leq 3) - P(X \leq 2)$ which is $0.857 - 0.677 = 0.18$ by looking in the table under $\lambda = 2$.

3. If approximately 2 percent of a town of 200 people are left handed, find the probability that exctly five people are left handed.

Soln: $\lambda = 200(0.02) = 4$. Using the table, $P(X = 5) = P(X \leq 5) - P(X \leq 4) = 0.785 - 0.629 = 0.156$.

4. (10 points) Given that the scores of 20 students in a test are 65,27,70,60,67,48,47,100,35,58,53,70,27,63,80,48,67,73,40 and 60 make a table dividing the data into 8 classes, showing the relative frequencies and class marks. Find the sample mean and the median.

Soln: Sample mean is the sum of values divided by 20 which equals 57.9. The median is the value at the halfway point when the data is arranged in order and you can see that it is 60.

5. Find the value of c such that $f(x)$ is the p.d.f of a continuous random variable X if f is given by $f(x) = x^3/4$, $0 < x < c$, and then find the distribution function $F(x)$ and the mean.

Soln:In order that $f(x)$ is a valid p.d.f we need that $\int_0^c f(x)dx = 1$ which means $[\frac{x^4}{16}]_0^c = \frac{c^4}{16} - 0 = 1$ which gives that $c = 2$. The distribution function is $F(x) = \int_0^x f(t)dt = x^4/16$. The mean is $\int_0^c xf(x)dx = \int_0^2 \frac{x^4}{4}dx = 2^5/20 = 8/5 = 1.6$.