Fall 09, Quiz 3, Probability and Statistics I Math 189-1 Sitaraman
10/5/09 Howard University Mathematics

SOLUTIONS

1.Check whether the events A & B are independent:
$P(A) = 0.25, P(B) = 0.5, P(A \cap B) = 0.124999.$

Soln: $P(A)P(B) = 0.125$ and does not equal $P(A \cap B)$. So the two events are not independent of each other.

2. A card is drawn from a deck of 52 cards, then the deck is reshuffled and another card is drawn from the same 52, and so on until 5 cards are drawn. Find the probability of getting 2,3,4,5,6 in any order. Use the product rule for independent events.

Soln: A given set of 2,3,4,5,6 (say 23456) can be obtained in $(4/52)^5 = \frac{1}{13^5}$ by multiplication for probabilities of each drawing because the probabilities are independent and each of the five numbers has a 4/52 or 1/13 chance. Now there are $5P_5 = 5! = 120$ ways in which you can get these five numbers (23456, 32546, 62345, etc.,). So probability of getting 2,3,4,5 or 6 is $\frac{120}{13^5}$.

3. Let S,C,D,H represent the event that a card is a spade, a club, a diamond or a heart repectively. Let A represent the event that a card is an ace. Prove that $P(A) = P(A/S)P(S) + P(A/C)P(C) + P(A/D)P(D) + P(A/H)P(H)$ using set theory as well as numerically.

Soln: Since S,C,H,D are mutually exclusive events, $A = A \cap S + A \cap C + A \cap D + A \cap H$ and thus $P(A) = P(A \cap S) + P(A \cap C) + P(A \cap D) + P(A \cap H)$ and this is equal to $P(A/S)P(S) + P(A/C)P(C) + P(A/D)P(D) + P(A/H)P(H)$ using definiton of conditional probability. Numerically, $P(A) = 1/13$ and $P(A/S) = 1/13$, $P(S) = 1/4, P(A/D) = 1/13$, $P(D) = 1/4, P(A/C) = 1/13, P(C) = 1/4, P(A/H) = 1/13, P(H) = 1/4$. Plugging thee into the equation, we get $1/13 = (1/13)(1/4) + (1/13)(1/4) + (1/13)(1/4) + (1/13)(1/4)$ which is true.

4. Of the 100 people in a town 60 are democrats, 20 are republicans and 20 are independents. Each of them voted either for Obama or McCain. 80 percent of Democrats, 50 percent of independents and 20 percent of Republicans voted for Obama. Given that a person voted for Obama,

find that he or she is a Republican using Bayes' theorem. Explain how you could have obtained this without the theorem.

Soln: Using Bayes' theorem, we get

$$P(R/O) = \frac{P(O/R)P(R)}{P(O/R)P(R) + P(O/D)P(D) + P(O/I)P(I)}$$

$$= \frac{(0.2)(0.2)}{(0.2)(0.2) + (0.8)(0.6) + (0.5)(0.2)} =$$

$$= 0.04/(0.04 + 0.48 + 0.1) = 0.04/0.62 = 4/62 = 0.0645.$$

Without Bayes' theorem, we could have simply counted the number of people who voted for Obama, etc., Notice that it amounts to the same thing.

5. A coin is tossed four times. Let X denote the random variable that counts the number of heads. Describe the domain of X and its probability mass function.

Soln:Domain is $X = 0, 1, 2, 3, 4$. The p.m.f is $f(x) = \frac{4C_x}{2^4}$.

6. Describe the probability mass function (p.m.f) of the random variable X that counts the number of women in a committee of 7 chosen from a group of 5 men and 6 women. Compute $f(2)$.

Soln: $f(x) = \frac{6C_x \times 5C_{7-x}}{11C_7}$, $x = 2, 3, 4, 5, 6$
$f(2) = \frac{6C_2 \times 5C_5}{11C_7} = 1/22.$