

Instructions:

PLEASE PROVIDE STEP BY STEP EXPLANATIONS

Time Limit 30 minutes

Please read the questions carefully before answering

Each problem 5 points unless otherwise stated.

1. Find the number of ways that a committee of 5 can be chosen from a group of 10 people.

$$\text{Soln: } {}_{10}C_5 = \frac{10!}{5!5!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 252.$$

2. Find the number of ways that you can arrange 3 red balls and 4 white balls in a row. Note: The balls are indistinguishable.

Soln: This is the number of distinguishable permutations of 7 things where 3 are of one kind and 4 another kind. The answer is $\frac{7!}{4!3!} = 35$.

3. Find the probability that in a hand of five cards out of fifty two cards, you would get 4 spades and a diamond.

Soln: Since there are 13 spades, you can choose the four spades in ${}_{13}C_4 = \frac{13!}{(9!4!)} = \frac{(13 \cdot 12 \cdot 11 \cdot 10)}{(24)} = 715$ ways. The diamond can be chosen in 13 ways. So total number of ways to pick four spades and a diamond is $(715)(13) = 9295$ ways. Total number of ways to choose 5 cards is ${}_{52}C_5 = 2598960$ ways. So the probability of this is $9295/2598960 = 0.00357643$.

4. Find the probability in a group of 5 people all would be democrats if the possibilities are that they are either democrats, independents or republicans.

Soln: Total number of ways to pick 5 people where each could be of 3 types is the number of unordered samples of size $r = 5$ among $n = 3$ things where order is irrelevant and is given by $(3 - 1 + 5)C_5 = 7C_5 = 21$ ways. Number of ways in which all are democrats is just 1. So probability is $1/21$.

5. If $P(A) = 0.5$ and $P(A \cap B) = 0.2$ find $P(B/A)$.

$$\text{Soln: This is just } P(A \cap B)/P(A) = 0.2/0.5 = 2/5$$

6. Find the probability that, given that five spades are chosen, one is a King?

Soln: Probability of choosing five spades is $P(5S) = 13C_5/52C_5$. Probability that a King is chosen among five spades is $P(K \cap 5S) = (1 \times 12C_4)/52C_5$ where you choose the King of spades and then the other four cards out of the remaining 12 spades that are non-Kings. So probability of 1 King given 5 spades are chosen is $P(K/5S) = P(K \cap 5S)/P(5S) = (12C_4)/13C_5 = 5/13 = 0.385$.