

Howard University Math Department

Instructions:

PLEASE PROVIDE STEP BY STEP EXPLANATIONS

WRITING ONLY ANSWERS WILL NOT GET FULL CREDIT

Time Limit 30 minutes

Please read the questions carefully before answering

1. (a) (6 points) Show that the following is an equivalence relation on the rational numbers, written such that the numerator and denominator have no common factors :

$$xRy \iff x, y \text{ have the same denominator.}$$

NOTE: Assume that the denominator of 0 is 1 (so $0 = 0/1$).

Example for no common factors : $2/4$ is written as $1/2$ after cancelling out the 2.

- (b) (9 points) Find the equivalence class of 2, 1.2 and $1/4$ under the above relation.

2. (15 points) Given the sequence $s_n = 5, 8, 11, 14, \dots, 3n + 2, \dots$ answer the following:

(a) Is it increasing, decreasing, nonincreasing or nondecreasing? (could be more than one).

(b) Find $\sum_{n=1}^4 s_n$. (c) Find the formula for the k -th term of the subsequence

$s_2, s_4, s_6, \dots$ (with even indices only) considered as a new sequence $a_1, a_2, a_3, \dots$

(So $a_1 = s_2, a_2 = s_4, \dots$)

3. (Extra credit) (8 points) Find the partition of the rational numbers created by the equivalence relation of problem 1.

(7 points) What kind of sequence is $5, 8, 11, 14, \dots$? what would be the formula for the n -th term if it started with 8? (when it starts with 5 it is $s_n = 3n + 2$).