9/26/2025 Fall 2025, Discrete Structures Midterm Test 1 v1 Solutions Sitaraman

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1. (20 points) Write the negative, converse and contrapositive of the following statement: "If you speak the truth you will be free of fear." Let p be the statement that you speak the truth and q be the statement that you will be free from fear.

   ALSO: write them in symbols.

   Solution:

   Symbolically the statement is $p \implies q$.

   Negative is "You speak the truth and you will not be free." $p \implies \neg q$.

   Converse is " If you are free from fear then you must be speaking the truth." $q \implies p$.

   Contrapositive is " If you are not free from fear then you are not speaking the truth." $\neg q \implies \neg p$.

2. (15 points) Write the following statement and its negative in symbols. Denote the set of natural numbers by $\mathbb{N}$.

   "For every natural number n, if $n > 1$ then $n^2 > 1$."

   Solution:

   Statement: $\forall n \in \mathbb{N}, n > 1 \implies n^2 > 1$.

   Negative: $\exists n \in \mathbb{N}, n > 1 \implies n^2 \leq 1$.

3. (15 points) Prove using the contrapositive: If $x+y+z > 30$ then one of x, y, z is bigger than 10.

   Solution:

   Proof: Assume opposite: all are less than or equal to 10. Then $x+y+z \leq 10+10+10 = 30$. So opposite of first statement is true.

4. (15 points) Prove using cases: If $x + y$ is odd, then x or y is odd.

   There are 4 cases: x odd, y odd ; x even, y odd ; x odd, y even ; x even , y even.

   Check that only the second and third cases work.

   NOTE: it doesn't say that x or y is odd means $x + y$ is odd. In other words, converse is not true. If both are odd then their sum will be even.

5. Given the universal set is $U = \mathbb{N}$ the set of natural numbers $\{1, 2, 3, ...\}$ and E the set of even natural numbers, O the set of odd natural numbers and S the set of squares of natural numbers $\{1, 4, 9, ...\}$, which of the following are true *for E, O , S, U*? Note that these are all infinite sets, so it is not always enough to check a few numbers. (5 points each)

   a) $E \cup O = U$; (b) $\overline{E \cap O} = E \cup O$; (c) $E - S = O$; (d) $E \cap S = S$.

   Solution:

   a) True, because every natural number is either even or odd.

   (b) is true by DeMorgan's law. Just note that complement of E is O and vice versa. You can also show that $E \cap O = \{\}$ so its complement is U which equals $E \cup O$.

   (c) False. Even if you take out all squares there will be a lot of even numbers left. There are even numbers that are not squares.

   (d) False. There are squares that are not even numbers, so in $E \cap S$ there will be elements that are not in S.

6. (15 points) Prove using induction:

$$1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}.$$

   Solution:

   REMEMBER: We are adding 1, 2, 3, ... all the way up to the n-th natural number, i.e, n.

   Proof for $n = 1 : 1 = 1(1 + 1)/2$.

   Assume formula works for n.

   Now prove for $n + 1$.

   Need to prove:

$$1 + 2 + 3 + ... + n + (n + 1) = \frac{(n + 1)((n + 1) + 1)}{2} = \frac{(n + 1)(n + 2)}{2}.$$

   Plugging in result for n we have

$$1 + 2 + 3 + ... + n + (n + 1) = \frac{n(n + 1)}{2} + n + 1 = (n + 1)\left(\frac{n}{2} + 1\right)$$
$$= (n + 1)\frac{n + 2}{2} = \frac{(n + 1)(n + 2)}{2}.$$

   The two sides are equal for $n + 1$ so we can go from n to $n + 1$ and the induction is complete.