9/26/2025 Fall 2025, Discrete Structures Midterm Test 1 v2 Solutions Sitaraman

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1. (20 points) Write the negative, converse and contrapositive of the following statement: "If you are kind people will like you." Let p be the statement that you are kind and q be the statement that people will like you.

   ALSO: write them in symbols.

   Solution:

   Symbolically the statement is $p \implies q$.

   Negative is "You are kind and people don't like you." $p \implies \neg q$.

   Converse is " If people like you then you must be kind." $q \implies p$.

   Contrapositive is " If you are not liked then you must be unkind." $\neg q \implies \neg p$.

2. (15 points) Write the following statement and its negative in symbols. Denote the set of natural numbers by $\mathbb{N}$.

   "For every natural number n, if $n > 1$ then $2n > 1$."

   Solution:

   Statement: $\forall n \in \mathbb{N}, n > 1 \implies 2n > 1$.

   Negative: $\exists n \in \mathbb{N}, n > 1 \implies 2n \leq 1$.

3. (15 points) Prove using the contrapositive: If $xyz > 64$ then one of x, y, z is bigger than 4.

   Solution:

   Proof: Assume opposite, namely all are less than or equal to 4. Then $xyz \leq 4 \times 4 \times 4 = 64$ which is the opposite of first statement.

4. (15 points) Prove using cases: If xy is even, then x or y is even.

   There are 4 cases: x odd, y odd ; x even, y odd ; x odd, y even ; x even , y even.

   Check that only the last 3 cases work. So one of x, y has to be even.

5. Given the universal set is $U = \mathbb{N}$ the set of natural numbers $\{1, 2, 3, ...\}$ and E the set of even natural numbers, O the set of odd natural numbers and S the set of 7 times natural numbers $\{7, 14, 21, ...\}$, which of the following are true *for E, O , S, U*? Note that these are all infinite sets, so it is not always enough to check a few numbers. (5 points each)

   a) $E \cup O = U$; (b) $\overline{E \cap O} = E \cup O$; (c) $E - S = O$; (d) $E \cap S = S$.

   Solution:

   a) True, because every natural number is either even or odd.

   (b) is true by DeMorgan's law. Just note that complement of E is O and vice versa. You can also show that $E \cap O = \{\}$ so its complement is U which equals $E \cup O$.

   (c) False. Even if you take out all multiples of 7 there will be still a lot of even numbers left. There are even numbers that are not multiples of 7.

   (d) False. There are multiples of 7 that are not even numbers, so in $E \cap S$ there will be elements that are not in S.

6. (15 points) Prove using induction:

$$1 + 4 + 7..... + (3n - 2) = \frac{n(3n - 1)}{2}.$$

   Solution:

   REMEMBER: We are adding 1, 4, 7, ... all the way up to the n-th number of the form "a multiple of 3 minus 2."

   Proof for $n = 1 :\ 1 = 1(3 - 1)/2$.

   Assume formula works for n.

   Now prove for $n + 1$.

   Need to prove:

$$1 + 4 + 7 + ... + (3n - 2) + (3(n + 1) - 2) = \frac{(n + 1)(3(n + 1) - 1)}{2} = \frac{(n + 1)(3n + 2)}{2}.$$

   Plugging in result for n we have

$$1{+}4{+}7{+}.....{+}(3n{-}2){+}3(n{+}1){-}2 = \frac{n(3n - 1)}{2}{+}3n{+}1 = \frac{3n^2 + 5n + 2}{2} = \frac{(n + 1)(3n + 2)}{2}.$$

   The two sides are equal for $n + 1$ so we can go from n to $n + 1$ and the induction is complete.