Howard University Math Department

Instructions:

PLEASE PROVIDE STEP BY STEP EXPLANATIONS

WRITING ONLY ANSWERS WILL NOT GET FULL CREDIT

Time Limit 30 minutes

Please read the questions carefully before answering

1. (20 points) Write the negative, converse and contrapositive of the following statement: "If you speak the truth you will be free of fear." Let p be the statement that you speak the truth and q be the statement that you will be free from fear.

   ALSO: write them in symbols.

2. (15 points) Write the following statement and its negative in symbols. Denote the set of natural numbers by $\mathbb{N}$.

   "For every natural number n, if $n > 1$ then $n^2 > 1$."

3. (15 points) Prove using the contrapositive: If $x + y + z > 30$ then one of x, y, z is bigger than 10.

4. (15 points) Prove using cases: If $x + y$ is odd, then x or y is odd.

5. Given the universal set is $U = \mathbb{N}$ the set of natural numbers $\{1, 2, 3, ...\}$ and E the set of even natural numbers, O the set of odd natural numbers and S the set of squares of natural numbers $\{1, 4, 9, ...\}$, which of the following are true *for E, O , S, U* ? Note that these are all infinite sets, so it is not always enough to check a few numbers. (5 points each)

   a) $E \cup O = U$; (b) $\overline{E \cap O} = E \cup O$; (c) $E - S = O$; (d) $E \cap S = S$.

6. (15 points) Prove using induction:

$$1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}.$$