

DISCRETE STRUCTURES 9/5/25 CLASSWORK

1. Write in symbols the following statements, letting $P(x)$ denote “ x is an accountant ” and $Q(x)$ denote “ x owns a Porsche.”

(1.4.48) All accountants own Porsches.

(1.4.51) Someone who owns a Porsche is an accountant.

Answer:

a) (1.5.48) $\forall x (P(x) \Rightarrow Q(x))$

b) (1.5.51) $\exists x (Q(x) \wedge P(x))$

2. Write in words the following statements, letting $P(x)$ denote “ x is a professional athlete ” and $Q(x)$ denote “ x plays soccer.”

a) (1.5.39) $\forall x (P(x) \Rightarrow Q(x))$. Every athlete is a soccer player

b) (1.5.46) $\exists x (P(x) \wedge Q(x))$. Someone who is an athlete is also a soccer player.

c) a) (1.5.42) $\exists x (Q(x) \Rightarrow P(x))$ There is a person who is an athlete if they are a soccer player.

Alternatively, “Some person can be an athlete if they play soccer.”

3. (1.5.53) Determine the truth value of the following statement. The domain of discourse is \mathbf{R} : $\forall x (x^2 > x)$ What is its negative?

Statement is false. It says that the square of every real number is bigger than that number.

Proof by counterexample (one is enough, but we give more):

$$1^2 = 1, 0.5^2 < 0.5, 0^2 = 0, 0.1^2 < 0.1, \dots$$

Its negative is : “Some real numbers are less than or equal to their squares.”

In symbols, $\exists x, x^2 \leq x$. (using DeMorgan’s law)