DISCRETE STRUCTURES 9/29/25 CLASSWORK

QUIZ 4 ON MONDAY 10/6

More details and study guide on update page.

PROOF BY INDUCTION

EXAMPLE 1: (2.4, 1) Prove by induction:

$$1 + 3 + 5 + \ldots + (2n - 1) = n^2$$

The sum of the first n odd numbers equals n^2.

Note that the n-th odd number is $2n - 1$.

Putting $n = 1,2,3, \ldots$ in $2n - 1$ we get 1, 3, 5,....

Prove for $n = 1$: $1 = 1^2$ (only one odd number and it is 1).

Assume it is true for $n = k$.

$1 + 3 + 5 + \ldots + (2k - 1) = k^2$

Use this as a stepping stone for the sum of the $k + 1$ odd numbers.

Since we already have k odd numbers on the left, we just need to add one more, namely $2(k + 1) - 1 = 2k + 1$ which is the $k + 1$-th odd number. (Or just add 2 to $2k - 1$ because odd numbers differ by 2).

$1 + 3 + 5 + \ldots + (2k - 1) + (2k + 1) = k^2 + (2k + 1) = (k + 1)^2$

So we have shown that, if the sum of the first k odd numbers obeys the formula, then the sum of the first $k + 1$ numbers obeys the same formula.

This means we can go from 1 to 2, then 2 to 3, then 3 to 4 and so on ad infinitum, and the formula will hold.

EXAMPLE 2: (2.4, 16) $2^n \geq n^2, n \geq 4$.

For $n = 4, 2^4 = 16 \geq 4^2 = 16$. **Note that we started from 4, not 1.** That is because we are only asked to do it for numbers bigger than or equal to 4. In fact it is not true for 3.

Now assume for n and *use that as a stepping stone* for $n + 1$.

Using $2^n \geq n^2$ we want to show $2^{n+1} \geq (n+1)^2$.

$2^{n+1} = 2^n + 2^n \geq n^2 + n^2$

How to show this is $\geq (n+1)^2 = n^2 + 2n + 1$?

Will be done if we can show $n^2 \geq 2n + 1, n \geq 4$.

One way : $n^2 \geq 2n + 1 \rightarrow (n^2 - 2n) \geq 1 \rightarrow n(n-2) \geq 1$

But $n(n-2) \geq 2$if n is bigger than 3.

Example 3: (2.4, 23) $11^n - 6$ is divisible by 5, for all $n \geq 1$.

For $n = 1$, we see $11 - 6 = 5$, so it is true that it is divisible by 5.

Now assume for n and *use that as a stepping stone* for $n + 1$.

$11^{n+1} - 6 = (11^n \times 11) - 6$

Now use that 5 divides $11^n - 6$ to get $11^n = 6 + 5k$ for some k and plug this in above expression.

Get $(11^n \times 11) - 6 = ((6 + 5k) \times 11) - 6 = ((6 \times 11) + (5k \times 11)) - 6$

$= (6 \times 11) - 6 + (5k \times 11) = (6 \times 10) + (5k \times 11) = 5 \times (12 + 11k)$

So we have proved that $11^{n+1} - 6$ is divisible by 5.