DISCRETE STRUCTURES 9/10/25 CLASSWORK

SUGGESTIONS FOR STUDYING:

1. Participate in class, ask questions, take notes
2. Try practice problems posted on update page (and other problems) from textbook.
3. Talk to me during office hours.
4. Get help from tutoring. Schedule on update page.

NESTED QUANTIFIERS

EXAMPLE: (1.6, 34) Domain is people, L(x,y) means "x loves y"

"Someone loves everybody" in symbols : $\exists x \forall y \; (L(x,y))$

This could be true : There are people who love everyone.

Negative of this is "There is no one who loves everybody"

or "For every person there is someone they do not love"

[Think of DeMorgan's law for an infinite set of and statements]

In symbols, $\forall x \; \exists y (\neg L(x,y))$

35. Everybody loves everybody

$\forall x \; \forall y \; (L(x,y))$

Negative: There is someone who doesn't love someone

$\exists x \; \exists y \; (\neg L(x,y))$

37. Everybody loves somebody

In symbols, $\forall x \; \exists y \; (L(x,y))$

Negative: There is someone who doesn't love anybody

$\exists x \; \forall y \; (\neg L(x,y))$

CONTINUED NEXT PAGE

Determine the truth value of the following and also write the negative:

Domain of discourse is **R** x **R**

48. $\forall x\ \forall y\ (x^2 < y + 1)$

This is false. Counter: x = 2, y = 1. The negative would be "there is some x and some y for which this is false." In symbols, $\exists x\ \exists y\ (x^2 \geq y + 1)$.

63. $\forall x\ \exists y\ ((x < y) \rightarrow (x^2 < y^2)$

This is true. Need to PROVE this for any two real numbers x and y.

Proof: Given any x, choose y > |x|. Then y > |x| because |x| $\geq$ x.

Also, if x, y are positive, then x < y means $x^2 < y^2$.

Proof: Say y = x+k, with k > 0. Then y = x^2+2kx+k^2 > x^2

because it is x^2 plus a positive number 2kx+x^2.

65. $\exists x\ \exists y\ ((x < y) \rightarrow (x^2 < y^2)$

PROBLEMS ON PROOF GIVEN IN CLASS TODAY

1. Show that if $x + y + z \geq 3,\ then\ x \geq 1 \vee y \geq 1 \vee z \geq 1$.

Proof by contrapositive: Assume second statement is false.

Then $x < 1 \wedge y < 1 \wedge z < 1. But\ then\ x + y + z < 3$.

So we proved the negative of the first statement and

thus the contrapositive is true, so given statement is also true.

2. Show that if $xy < 2\ then\ x < \sqrt{2}\ OR\ \ y < \sqrt{2}$.

Use proof by contrapositive also. Similar to 1.