Discrete Structures QUIZ I ON WEDNESDAY!

8/22/25 Class notes and exercises

Textbook 1.1.34

Show that the sets A = {1,2,3} and B = $\{n \mid n \in Z^+ \; and \; n^2 < 10\}$ are equal.

How to show: Show that A is contained in B and B is contained in A.

Note: N = Natural numbers, Z = Integers, Q = Rational numbers, R = Real numbers.

A is contained in B : Just check that 1^2 < 10, 2^2 < 10, etc.,

B is contained in A : If n is a positive *integer such that* $n^2 < 10$ *then n is less than or equal to 3.*

Class Question 1: What is the complement of A = { People who know they know}

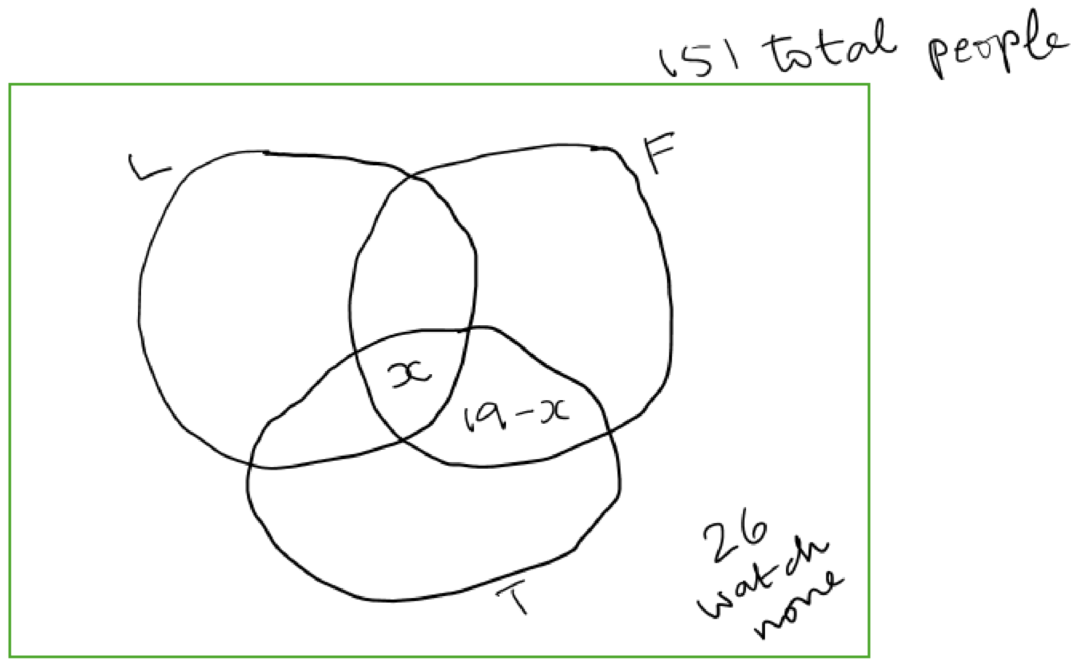
Complement of A = All people outside of A

= People either in B, C, or D = $B \cup C \cup D$

Textbook Qn 1.1.66

A television poll of 151 persons found that 68 watched “Law and Disorder”; 61 watched “25”; 52 watched “The Tenors”; 16 watched both “Law and Disorder” and “25”; 25 watched both “Law and Disorder” and “The Tenors”; 19 watched both “25” and “The Tenors”; and 26 watched none of these shows. How many watched all three?

Answer next page



There are 151 people and 26 watch none.

So 151-26 = 125 watch either L (law and disorder) or F (“25”) or T (“Tenors”).

So $125 = L \cup F \cup T$ and this is basically the people inside the circles.

We need to figure out how many watch all three shows and we call that x.

We will solve for x by writing 125 in another way, using x.

From the picture, you can see that one way to get to 125 is:

$$L \cup F \cup T = 125$$

$=$ Number in L + Number in (F-L)

+ Number in (T – (those in either $L \cap T$ or $F \cap T$))

= 68 + (61 – 16) + (52 – (25 +(19- x))) = 68 + 45 + 27 – 19 + x = 121 + x

So we get 125 = 121 + x and solving that we get $x = 4$.

So 4 people watch all 3 shows.

NOTE: In the above, Number in L + Number in (F-L) actually equals the number in $L \cup F$. We also have: Number in $F - L$ = Number in F – Number in $F \cap L$.

More generally, we can use the following formulae (for any set A, we denote the number of elements in A by |A|) :

$$|AUB| = |A| + |B - A| = |A| + |B| - |A \cap B|$$

$$|A \cup B \cup C| = |A| + |B| + |C| - (|A \cap B| + |A \cap C| + |B \cap C|) + |A \cap B \cap C|.$$

The last formula is an example of an “inclusion-exclusion” formula.

We add up the number of elements of each set, then subtract the number of elements in two of them taken at a time, and then add the number of elements in three of them taken at a time, and so on. Basically we try to subtract the ones we counted twice when we added the number of elements in each set. But then when we do that we subtract the ones in all three of them, twice. So we have to add the number of elements in all three of them.