Please go to Update page and Course page to see information about class and to keep up to date. The links are in Canvas and also on http://nature-lover.net/math. You can see old notes from fall 2017 etc at this website. It will help you prepare for class.

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NEXT QUIZ : FRIDAY, 10/31, CHAPTER 6, SECTIONS 1 AND (POSSIBLY) 2.

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Today: Counting methods

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Review from last class:

MULTIPLICATION PRINCIPLE:

Number of ways to do something that involves m INDEPENDENT steps each of which can be done in n_1, n_2, n_3, ..., n_m ways is $n_1 n_2 \ldots n_m$

ADDITION PRINCIPLE:

If X_1, X_2,….,X_m are disjoint sets with n_1, n_2, n_3, …., n_m elements respectively, then number of total elements is $n_1 + n_2 + \ldots + n_m$

If they are not disjoint, use **inclusion exclusion principle**:

For example: $|X \cup Y| = |X|+|Y| - |X \cap Y|$ where $|X|$ means number of elements in X, etc.,
Similarly $|X \cup Y \cup Z| = |X|+|Y| + |Z| - |X \cap Y| - |X \cap Z| - |Y \cap Z| + |X \cap Y \cap Z|$

Problem 7: You (Y) and your best 4 friends are sitting and eating on one side of a table.
Say they are A, B, C, D.
How many ways can you arrange all of you in 4 chairs?

ABCDY, ACBDY, etc.,

Can count it as 5 x 4 x 3 x 2 = 120.

In general 1 x 2 x 3 x…x (n-1) x n is denoted by n!
So 5! = 5 x 4 x 3 x 2 x 1 = 120.

5 x 4 x 3 = 5!/2!

This pattern is called **permutation**: Arranging 5 things in 4 places.
First place : 5 choices, and for each choice of first, have 4 choices for
second place, 3 for 3rd and 2 for 4th. More about permutations later.

So in general, nPk = number of permutations of n things taken k at a time is n! /(n-k)!
For example, 5P3 = 5!/(5-3)! = 5! / 2! = 5 x 4 x 3 = 60.

Counting methods summary :

1) order matters: (a) repetition allowed (b) repetition not allowed.

1(b) is also called **permutations.**

Formula for 1a: arranging n things in k places, with repetition, is n^k.
Formula for 1b: arranging n things in k places, without repetition, is nPk defined above.

2) What if order does NOT matter?

Today we will look at **combinations**: counting number of ways to select k things out of n, without regard for order.

Example 1: Number of ways to **choose or select** 3 people for a committee from a group of 5, where they are all equal (no ranks or officers).

It would be smaller than 60. Why? Because number of selections is a subset of number of arrangements.

It will be the same people, just that we don't count A, B, C differently from A, C, B and so on.

In fact,

(A, B, C) , (A, C, B), (B, C, A), (B, A, C), (C, B, A) and (C, A, B) would all count as one!

This is arranging 3 people in 3 places and it is given by

$$3P_3 = \frac{3!}{(3-3)!} = \frac{3!}{0!} = \frac{3!}{1!} = 6.$$

So answer is 60/ 6 = 10 selections of 3 people from 5.

Reason: If we have a team of 3 people, they can be arranged in the 3 positions, pitcher, catcher and batter in 6 ways.

So all the 60 teams can be arranged in 10 blocks of 6 teams, each with same 3 people

COMBINATIONS of k from n

= number of selections of k from n = number of arrangements of n in k

places divided by arrangements of k people of k places.

= nCk = n!/(n-k)!k! = nPk/k!
5C3 = 5! / (3! X 2!) = 120 / (6 x 2) = 10

Example 2: Smaller example:

say you want to pick 2 from 3.

(a, b) or (a, c) or (b, c) - three teams with different players.

Suppose want to also look at number of teams, with order of players. (who plays front, who is in back?)

Then we get (a, b) and (b, a), or (a, c) or (c, a) or (b, c) or (c, b). So totally 6.

The 6 arrangments are the permutations. Get them using 3P2 = 3!/1! = 6

The 3 teams (with different players) are the combinations.

So once you pick the number of selections, (combinations), we can get the number of arrangements (permutations), by multiplying by

k! (here 2! = 2 ways to arrange 2 people).

3P3 = 3!/(3 − 3)! = 3!/1 = 3! = 6

So 3C2 x 2! = 3P2 or 3C2 = 3P2 / 2!

In general, nCk = nPk / k! = $\frac{n!}{(n-k)!k!}$

Example 3 (6.2 Problem 33-37) A club has 6 men and 6 women. How many ways to select the following:

(33) Committee of 5.
(34) Committee of 3 men and 4 women.
(35) Committee of 4 with at least one woman.

(36) Committee of 4 with at most one man.
(37) Committee of 4 with people of both genders?

ANSWERS:

(33) Combinations of 5 out of 12 = 12!/(7! 5!)
= (12 x 11 x 10 x 9 x 8)/(5 x 4 x 3 x 2 x 1)
= 4 x 11 x 2 x 3 x 4 = 44 x 24 = 1056.

(34) Use multiplication after finding 6C3 and 6C4 (number of ways to pick 3 men from 6 and 4 women from 6)
6C3 = 6! /(3! 3!) = (6 x 5 x 4) / (3 x 2 x 1) = 20.
6C4 = 6!/(4! x 2!) = (6 x 5) / 2 = 15
Total ways to pick 3 men and 4 women equals 15 x 20 = 300.

(35) "At least woman" is opposite of "no women" in committee.
So number of ways to pick 4 with at least one
= Total number of committees minus number of committees with no women
= Number of ways to pick 4 from 12 minus number of ways to pick 4 from only the 6 men
= 12C4 – 6C4 = 495 – 15 = 480 ways.