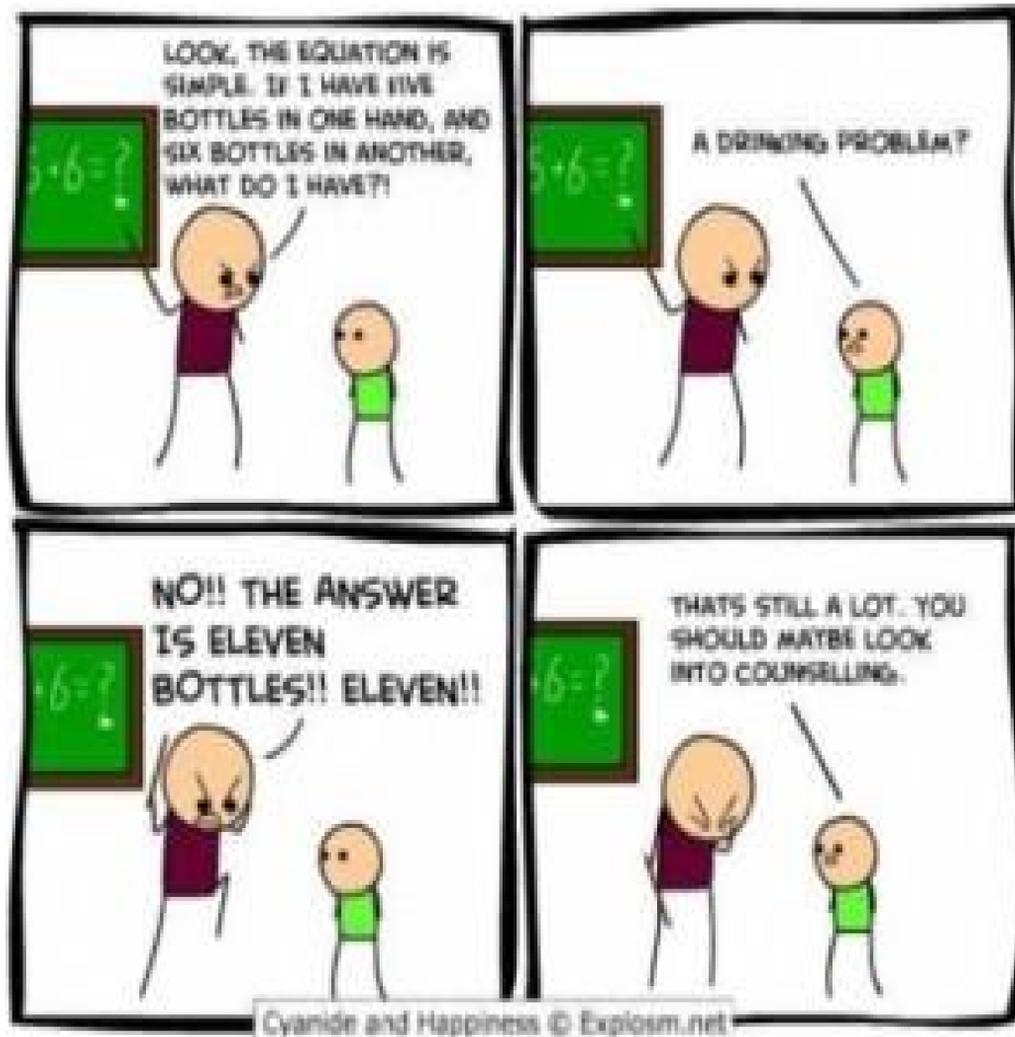


Chapter I. Section 2.

2-7-2018 class notes

APPLICATIONS OF LINEAR AND RATIONAL EQUATIONS



HOW TO SOLVE WORD PROBLEMS?

IDENTIFY THE VARIABLE

WRITE EVERYTHING IN TERMS OF THAT VARIABLE

SET UP EQUATION USING GIVEN INFORMATION

SOLVE EQUATION

CHECK ANSWER.

EXAMPLE:

Franz borrowed a total of \$10,000. Part of the money was borrowed from a bank that charged 5.5% simple interest. The rest of the money was borrowed from a friend to whom Franz paid 2.5% simple interest. Franz paid his friend back after 9 months (0.75 yr) and paid the bank after 2 yr. If the total amount Franz paid in interest was \$735, how much did he borrow from each source?

FORMULA FOR SIMPLE INTEREST: $I = Prt$

(I is interest, P is principal, r is interest rate per year, t is number of years)

STEP I: IDENTIFY VARIABLE

Usually this is obvious from the problem.

Here you have a choice:

Since there are two unknowns we could let either one be our variable.

Let x be the amount borrowed from the bank.

STEP II: WRITE ALL UNKNOWNNS IN TERMS OF THAT VARIABLE.

Since totally there is 10,000 available,
the amount borrowed from friend is $10000 - x$.

STEP III: SET UP EQUATION.

BASICALLY, WE TRANSLATE GIVEN INFORMATION INTO SYMBOLS

Total interest = 735 = Interest paid to Bank + Interest paid to friend.

Now using formula, with $P = x$ for bank and $P = 10000 - x$ for friend,

$$735 = x(0.055)(2) + (10000 - x)(0.025)(0.75) = 0.11x + 187.5 - 0.01875x$$

STEP IV: SOLVE EQUATION

$$\begin{aligned} \text{We have } 0.11x - 0.01875x &= 735 - 187.5 \implies 0.09125x = 547.5 \\ &\implies x = 6000. \end{aligned}$$

So he paid 6000 to the bank and 4000 to his friend.

STEP V: CHECK ANSWER (IF YOU WISH)

6000 times 0.055 times 2 is 660.

4000 times 0.025 times 0.75 is 75.

$660 + 75 = 735$. SO our answer is correct.

A PRACTICE PROBLEM

Rene drove from Miami to Orlando, a total distance of 240 miles.

He drove for 1 hr in city traffic and for 3 hr on the highway.

If his average speed on the highway was 20 mph faster than his speed in the city, determine his average speed driving in the city and his average speed driving on the highway.

Formula: Distance = Speed times Time

HOW TO SOLVE WORD PROBLEMS?

IDENTIFY THE VARIABLE

WRITE EVERYTHING IN TERMS OF THAT VARIABLE

SET UP EQUATION USING GIVEN INFORMATION

SOLVE EQUATION

CHECK ANSWER.

STEP I: IDENTIFY VARIABLE

Let x be the average speed in the city.

STEP II: WRITE ALL UNKNOWNNS IN TERMS OF THAT VARIABLE.

Since average speed in highway is 20 mph more, it equals is $x + 20$.

STEP III: SET UP EQUATION.

BASICALLY, WE TRANSLATE GIVEN INFORMATION INTO SYMBOLS

Total distance = 240 = Distance driven in city + Distance driven on highway.

Now using formula, distance = speed x time,

$$240 = x(1) + (x + 20)(3)$$

STEP IV: SOLVE EQUATION

We have $x + 3x + 60 = 240 \implies 4x = 180$

$$\implies x = 45.$$

So he paid drove at 45 mph in city and 65 mph on highway.

STEP V: CHECK ANSWER (IF YOU WISH)

65 times 3 is 195.

45 times 1 times 45.

$195 + 45 = 240$. So our answer is correct.

Quote from an interview with Prof. Ed Frenkel, author of “Love and Math”

Our mathematical illiteracy can have disastrous consequences. Case in point: Frenkel blames the global economic crisis of 2008-09 on inadequate mathematical models used by bankers and traders to predict the financial markets. “We should all have access to the mathematical knowledge and tools needed to protect us from arbitrary decisions made by the powerful few in an increasingly math-driven world,” writes Frenkel in his book, *Love and Math: The Heart of Hidden Reality*. “Where there is no mathematics, there is no freedom.”

For more go to <https://www.motherjones.com/environment/2014/02/inquiring-minds-edward-frenkel-math-doesnt-suck/>

PROBLEMS INVOLVING RATES OF WORK.

Similar to distance and speed problem, except here speed is the rate at which the work is done.

KEY IDEA:

Look at the amount of work each person can do in one unit of time.

Adding them, you get the rate at which they work together.

WARNING: If one person takes 1 hour, and the other person takes 2 hours, together they won't take 3 hours! Or even the average of 1 and 2.

They would in fact take *less than one hour* !!

FORMULA TO REMEMBER

If it takes T hours to do a job, in one hour you would do $1/T$ of the job.

Example: if you can mow the lawn in 2 hours, in one hour you would mow $1/2$ of the lawn.

Example:

Sheldon and Penny were awarded a contract to paint 16 offices in the new math building at a university. Once all the preparation work is complete, Sheldon can paint an office in 30 min and Penny can paint an office in 45 min.

How long would it take them to paint one office working together?

How long would it take them to paint all 16 offices?

First identify the variable. What is the unknown?

Let us call t the time it takes to paint one office, working together.

What are the rates at which they are working?

Sheldon takes 30 minutes for one office, so he paints two offices per hour.

Penny takes 45 minutes or $0.75 = 3/4$ hours for one office, so she would paint $1/0.75 = 1/(3/4) = 4/3$ or 1 and $1/3$ office in one hour.

So together they would work at the rate of $2 + (4/3) = 10/3 = 3 + (1/3)$ offices per hour.
 So to paint one office it would take $1/(10/3) = 3/10$ of an hour
 which is 60 times $3/10$ equals 18 minutes.
 To paint 16 offices it would take 16 times 18 or 288 minutes.

PRACTICE PROBLEM

How much 4% acid solution should be mixed with 200 mL of a 12% acid solution to make a 9% acid solution?

HINT: This is similar to the money problem we did earlier, except here we have two containers with different percentages of acid instead of two loans with different percentages of interest.

HOW TO SOLVE WORD PROBLEMS?

IDENTIFY THE VARIABLE

WRITE ALL UNKNOWNNS IN TERMS OF THAT VARIABLE

SET UP EQUATION USING GIVEN INFORMATION

SOLVE EQUATION

CHECK ANSWER.

ANSWERS BELOW

So together they would work at the rate of $2 + (4/3) = 10/3 = 3 + (1/3)$ offices per hour.
 So to paint one office it would take $1/(10/3) = 3/10$ of an hour
 which is 60 times $3/10$ equals 18 minutes.
 To paint 16 offices it would take 16 times 18 or 288 minutes.

PRACTICE PROBLEM

How much 4% acid solution should be mixed with 200 mL of a 12% acid solution to make a 9% acid solution?

HINT: This is similar to the money problem we did earlier, except here we have two containers with different percentages of acid instead of two loans with different percentages of interest.

HOW TO SOLVE WORD PROBLEMS?

IDENTIFY THE VARIABLE

WRITE ALL UNKNOWNNS IN TERMS OF THAT VARIABLE

SET UP EQUATION USING GIVEN INFORMATION

SOLVE EQUATION

CHECK ANSWER.

let x be amount of acid at 4%
 Amount of acid chemical in x mL
 $= .04x$
 " " in 200 mL = $.12(200) = 24$
 Total amount = $(200 + x)(.09)$
 Equation: $.04x + 24 = (200 + x)(.09)$
 $.04x + 24 = 18 + .09x$
 $\Rightarrow 6 = .05x$
 $\Rightarrow x = 6 / .05 = 120$ mL
 Check: 4% of 120 = 4.8
 $28.8 = 320 \times 9\%$? ✓