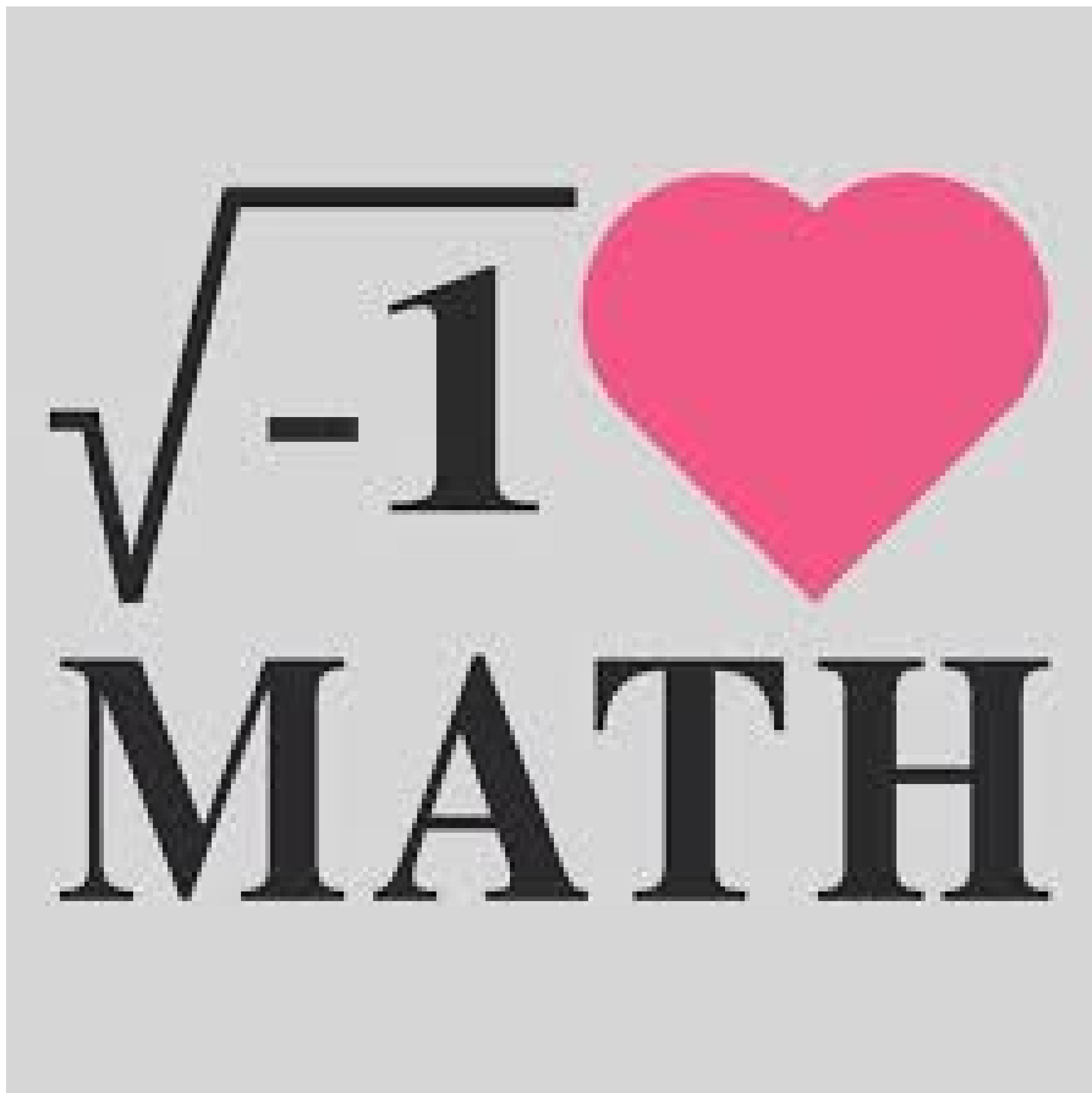
Chapter I. Section 3 and 4.

2-12-2018 class notes

SOLUTIONS OF QUADRATIC EQUATIONS ; IMAGINARY NUMBERS



WHY DO WE NEED IMAGINARY OR COMPLEX NUMBERS?

Just as we invented negative numbers to subtract bigger numbers from smaller
in other words to solve equations like $x+3 = 2$
We invented complex numbers in 16th century onwards
to find square roots of negative numbers
i.e, to solve equations like $x^2 + 1 = 0$.

So we say

$$\sqrt{-1} = i$$

NICE BONUS:
SQUARE ROOTS OF ALL NEGATIVE NUMBERS CAN BE WRITTEN USING i !!

Example: $\sqrt{-16} = \sqrt{16} \times \sqrt{-1} = 4i$.
Write the following using i :

1. $\sqrt{-23}$
2. $\sqrt{-20}$
3. $\sqrt{-75/18}$
4. $\sqrt{-\pi}$

ANSWERS

$$\sqrt{-23} = (\sqrt{23})i.$$

$$\sqrt{-20} = \sqrt{-1}\sqrt{4 \times 5} = 2\sqrt{5}\ i.$$

$$\sqrt{-75/18} = \sqrt{-1}\sqrt{25/6} = (5/\sqrt{6})i$$

$$\sqrt{-\pi} = \sqrt{\pi}\ i.$$

Okay, so we can solve any equation of the form $x^2 = r$
i.e, $x^2 - r = 0$. where r is any real number.

What about equations like $(x-1)^2 = -1$ **?**
This needs x such that $x - 1 = \pm\sqrt{-1} = \pm i$.

For this we invent numbers called *COMPLEX NUMBERS*.
This is any number of the form $a + bi$ where a, b are real numbers.
$a \equiv$ real part ; $bi \equiv$ imaginary part.

LATER: **ANY QUADRATIC EQUATION** $ax^2 + bx + c = 0$
can be written as $(Ax + B)^2 = C$. **!!**
So complex numbers can solve ANY quadratic equation,
just as real numbers can solve any linear equation.
AMAZING FACT: Complex numbers can actually solve
ANY POLYNOMIAL EQUATION OF ANY DEGREE!

OPERATIONS WITH COMPLEX NUMBERS
Similar to operating with real numbers, except
remember that $i^2 = -1$ **!**
Also, separate the real and imaginary parts when you are adding and
also after you have completed the calculations.

Examples:

1. $(13 - 4i) + (2 + 6i)$
2. $(2 + 3i)^3$
3. $\dfrac{1+i}{1-i}$ [Hint: Multiply by conjugates to "real"ize denominator]
4. $(\sqrt{-7} + 1)(\sqrt{-4} - 3)$
5. i^{63} [Hint: write 63 as 2n+1 for some n]

ANSWERS

1. $(13 - 4i) + (2 + 6i) = (13 + 2) + (-4 + 6)i = 15 + 2i$
2. $(2 + 3i)^3 = 2^3 + 3(2^2)(3i) + 3(2)(3i)^2 + (3i)^3 = 8 + 36i + 6(9i^2) + 27i^2(i)$
   $= 8 + 36i + 6(-9) + 27(-1)i = 8 + 36i - 54 - 27i = -46 + 9i$

3.

$$\frac{1+i}{1-i} = \frac{1+i}{1-i}\frac{(1+i)}{(1+i)} = \frac{1+2i+i^2}{1-i^2} = \frac{1+2i-1}{1-(-1)} = 2i/2 = i.$$

4. $(\sqrt{-7}+1)(\sqrt{-4}-3) = (1+\sqrt{7}i)(-3+2i) = -3+2i+(-3)(\sqrt{7})i+(2i)(\sqrt{7})i$
$= -3+2i-(3\sqrt{7})i+2\sqrt{7}(-1) = (-3-2\sqrt{7})+(2-3\sqrt{7})i$

5. $i^{63} = i^{2\times 31+1} = (i^{2\times 31})i = (i^2)^{31}i = (-1)^{31}i = -i$