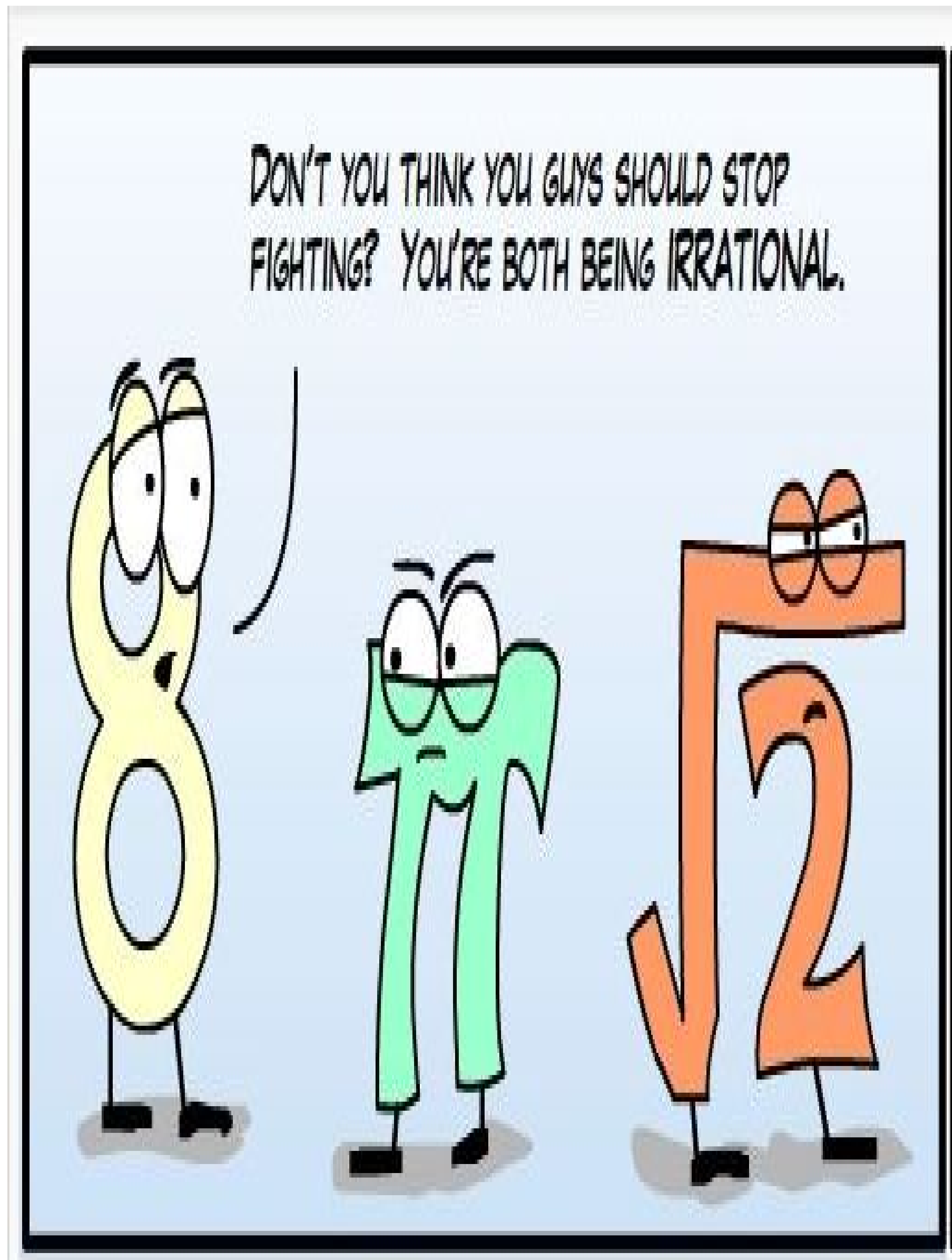
R6. More on Rational and Radical Expressions

1-31-2018 class notes



A rational expression is of the form

$$\frac{polynomial}{polynomial}$$

NOTE: The polynomial in the denominator cannot become zero when you plug in numbers for the variables if you want to get a real number value out of the rational expression.

Many of the properties of rational expressions are similar to those of rational numbers

Example:

$$\frac{x^2+3x-x^3}{x^2-4}$$

This expression has 2 and -2 as values for which x^2-4 is zero.
Therefore it has no value at 2 and -2.

2. Simplifying Rational Expressions

Just like with fractions:
1. First factor everything as much as you can
2. Cancel common terms, and add fractions with the same denominator.
3. If denominators are different, find least common denominator.
4. Multiply each numerator with "missing" term.
5. Once all denominators are equal, add numerators.
6. **Remember: Dividing by a fraction ≡ Multiplying by its reciprocal**

Remember how we added fractions:

$$\frac{3}{8}+\frac{5}{6}=\frac{3\times 3}{8\times 3}+\frac{5\times 4}{6\times 4}=\frac{9}{24}+\frac{20}{24}=\frac{9+20}{24}=\frac{29}{24}$$

The LCM was 24 ($48 = 8 \times 6$ would also work but it is bigger).
The "missing" factors are 3 and 4:
to get 24 in first denominator you need to multiply it by 3 ;
to get same in sedond denominator you need to multiply it by 4.
BTW, how do you find LCM of two numbers?
(See last page forexample LCM of 20 and 24;
basically you multiply all the prime factors to the highest degree)

EXAMPLE:

$$\frac{1}{x^2+xy}-\frac{2}{x^2-y^2}$$

1. Factor everything as much as possible:
$x^2+xy = x(x+y); x^2-y^2 = (x+y)(x-y)$.
2. Find LCM of denominators, since they are not equal.
LCM = product of all factors, with highest power for each factor.
In this case, LCM is $x(x+y)(x-y)$.
3. Multiply numerators by "missing" terms and subtract:

$$\frac{1\times(x-y)}{x(x+y)(x-y)}-\frac{2\times x}{x(x+y)(x-y)}=\frac{x-y-2x}{x(x+y)(x-y)}=-\frac{y+x}{x(x+y)(x-y)}.$$

Not done yet! We see that now $x+y$ is common to both numerator and denominator.
So we cancel it and get the answer as $-1/(x(x-y)) = -1/(x^2-xy)$.

Rationalizing denominator

To remove square-root from denominator, multiply and divide by *conjugate.*

Example:

$$\frac{1}{2-\sqrt{3}} = \frac{1 \times 2+\sqrt{3}}{2-\sqrt{3} \times 2+\sqrt{3}} = \frac{2+\sqrt{3}}{2^2-(\sqrt{3})^2} = \frac{2+\sqrt{3}}{4-3} = 2+\sqrt{3}.$$

SOME PRACTICE PROBLEMS

ANSWERS AT THE BOTTOM

Simplify the following, as much as you can.

1. $\dfrac{x^2-9}{x^2-4x-21}$

2. $\dfrac{15tu^5v}{3tu^2}$

3. $\dfrac{8x-3y}{x^3y^4}\times\dfrac{6xy^8}{24x-9y}$

ANSWERS BELOW

4. $\dfrac{4}{x^2+6x+5}-\dfrac{3}{x^2+7x+10}$

5. $\dfrac{8}{\sqrt{15}-\sqrt{11}}$

6. $\dfrac{\dfrac{1}{x+1}}{\dfrac{-5}{x^2-3x-4}+\dfrac{1}{x-4}}$

ANSWERS BELOW

SOME PRACTICE PROBLEMS

Simplify the following, as much as you can.

1. $\frac{x^2-9}{x^2-4x-21}$ $= \frac{(x+3)(x-3)}{x^2-7x+3x-21} = \frac{(x+3)(x-3)}{(x-7)(x+3)} = \frac{x-3}{x-7}$

Cancel like this because it is multiplication

2. $\frac{15tu^5v}{3tu^2}$ $= 5u^{5-2}v = 5u^3v$

3. $\frac{8x-3y}{x^3y^4} \times \frac{6xy^8}{24x-9y}$ $= \frac{8x-3y}{x^3y^4} \times \frac{6xy^8}{3(8x-3y)} = \frac{2y^{8-4}}{x^{3-1}}$

$= 2y^4/x^2$

4. $\frac{4}{x^2+6x+5} - \frac{3}{x^2+7x+10}$ $= \frac{4}{(x+5)(x+1)} - \frac{3}{(x+5)(x+2)}$

LCM of denominators $(x+5)(x+1)(x+2)$

$= \frac{4(x+2)}{LCM} - \frac{3(x+1)}{LCM}$

$= \frac{4x+8-3x-3}{LCM} = \frac{x+5}{LCM}$

$= \frac{1}{(x+1)(x+2)}$

5. $\frac{8}{\sqrt{15}-\sqrt{11}}$

$= \frac{8(\sqrt{15}+\sqrt{11})}{(\sqrt{15}-\sqrt{11})\times(\sqrt{15}+\sqrt{11})} = \frac{8(\sqrt{15}+\sqrt{11})}{15-11}$

$= 8(\sqrt{15}+\sqrt{11})/4$

$= 2(\sqrt{15}+\sqrt{11})$

6. $\frac{\frac{1}{x+1}}{\frac{-5}{x^2-3x-4}+\frac{1}{x-4}}$

$= \frac{\frac{1}{x+1}}{\frac{-5}{(x-4)(x+1)}+\frac{1}{x-4}} = \frac{\frac{1}{x+1}}{\frac{-5+1(x+1)}{(x-4)(x+1)}} = \frac{\frac{1}{x+1}}{\frac{x-4}{(x-4)(x-1)}} = \frac{1}{x+1}\times\frac{x-1}{1}$

$= \frac{x-1}{x+1}$

Break into prime factors

$24 = 2^3 \times 3$

$20 = 2^2 \times 5$

$LCM = 2^3 \times 3 \times 5$

$= 120$

= least common multiple of 20 & 24