R5 Factoring of Polynomials and Radical Expressions

1-29-2018 class notes

Factoring is just multiplication in reverse

METHODS OF FACTORING

1. Taking out common terms

Example 1: $x^2 + 3x - x^3 = x(x + 3 - x^2)$; $\quad y\sqrt{x} - x\sqrt{y} = \sqrt{x}\sqrt{y}(\sqrt{y} - \sqrt{x})$.

2. Factoring using grouping

$x^3 + x^2 + x + 1 = (x^3 + x^2) + (x + 1) = x^2(x + 1) + 1(x + 1) = (x^2 + 1)(x + 1)$.

$x^2+7x+10 = x^2+5x+2x+10 = (x^2+5x)+(2x+10) = x(x+5)+2(x+5) = (x+2)(x+5)$.

You can factor any *trinomial like this.*
1. Break down x coefficient into two whose product equals contant term times coefficient of x^2.
2. Factor using grouping as above

Example: $3x^2 + 5x - 2 = 3x^2 + 6x - 1x - 2 = 3x(x + 2) - 1(x + 2) = (3x - 1)(x + 2)$

3. Factoring using formulae

FORMULAE TO REMEMBER

$(A + B)^2 = A^2 + B^2 + 2AB$ (Square of Sum = Sum of Squares Plus Twice Product).

$(A + B)(A - B) = A^2 - B^2$ (Sum times Difference = Difference of Squares).

$(A + B)^3 = A^3 + 3A^2B + 3AB^2 + B^3$.

$(A + B)(A^2 - AB + B^2) = A^3 + B^3$

NOTE: In all of above, you can replace B with -B throughout and get corresponding formula.

EXAMPLES:

$(4x^2 - 16) = (2x)^2 - 4^2 = (2x - 4)(2x + 4)$.

$a^3 + 27 = (a + 3)(a^2 - 3a + 9)$

$y^6 - 8 = (y^2)^3 - 2^3 = (y^2 - 2)(y^4 + 2y^2 + 4)$.

(You can substitute $y^2 = x$ if you wish. Then it will be $x^3 - 2^3$).

$x^3 + 3x^2 + 3x + 1 = (x + 1)^3$.

$100 - 20z + z^2 = (10 - z)^2$.

SOME PRACTICE PROBLEMS

ANSWERS AT THE BOTTOM

Factor the following, as much as you can.

1. $x^2 - 16$

2. $7x^2 + 18x - 9$

3. $x^2 + 64$

ANSWERS BELOW

4. $4y^5 + 3y^3 + 4y^2 + 3.$

5. $x^3 - 1.$

5. $y^3 + 2 + \dfrac{1}{y^3}$

ANSWERS BELOW

SOME PRACTICE PROBLEMS

Factor the following, as much as you can.

1. $x^2 - 16$ $= x^2 - 4^2$ Use $A^2 - B^2 = (A+B)(A-B)$

$= (x+4)(x-4)$

2. $7x^2 + 18x - 9$ Look for m, n such that

$mn = 7x - 9$, $m+n = 18$

$m = 21$ $= 7x^2 + 21x - 3x - 9$

$n = -3$ $= 7x(x+3) - 3(x+3) = (7x-3)(x+3)$

3. $x^2 + 64$ $=$ NOT $(x+8)(x-8)$ $[= x^2 - 64]$

Cannot factor! (Sum of 2 squares)

4. $(4y^5 + 3y^3) + (4y^2 + 3)$

$= y^3(4y^2+3) + 1(4y^2+3)$

$= (4y^2+3)(y^3+1) = (4y^2+3)(y+1)(y^2-y+1)$

5. $x^3 - 1$. $= x^3 - 1^3$ Use $A^3 + B^3$ $\leftarrow y^3 + 1^3$

$A^3 - B^3$ formula $= (x-1)(x^2 + x + 1)$

5. $y^3 + 2 + \frac{1}{y^3}$ actually comes from $(A+B)^2 = A^2 + 2AB + B^2$

$A = \sqrt{y^3}$ $B = \frac{1}{\sqrt{y^3}}$ $(A+B)^2 = y^3 + 2\frac{\sqrt{y^3}}{\sqrt{y^3}} + \frac{1}{y^3} = y^3 + 2 + \frac{1}{y^3}$

ALSO $\frac{y^6 + 2y^3 + 1}{y^3} = \frac{(y^3)^2 + 2y^3 + 1^2}{y^3} = \left(\frac{y^3+1}{y^3}\right)^2 = \left(\frac{y^3+1}{\sqrt{y^3}}\right)^2$