R4 Addition and Multiplication of Polynomials and Radicals

1-26-2018 class notes

1. What is polynomial?

A polynomial in the variable x is a sum of (whole) powers of x.
The highest power is called the **degree** of the polynomial.
The numbers multiplying the powers of x are called **coefficients.**

EXAMPLES
$0.1x^2 + x - 1, 100, x, \pi - x^4, x - x^2 - x^3$ are polynomials.
What are their degrees and coefficients?
$\sqrt{x},\ 1/x$,etc., are not polynomials (the powers are fractions or negative.

OPERATIONS WITH POLYNOMIALS AND RADICAL / RATIONAL POWERS

ADDITION:
Gather like terms
Add coefficients

EXAMPLE:

$$(x^2 - 2x + \frac{\sqrt{x}}{2}) + (x^3 + 0.5x^2 - x + \sqrt{x}) = x^3 + (1 + 0.5)x^2 + (-2 - 1)x + (\frac{1}{2} + 1)\sqrt{x}$$

$$= x^3 + 1.5x^2 - 3x + 1.5\sqrt{x}.$$

MULTIPLICATION
Multiply each term of one expression with each term of other
For each product, add powers of x and multiply coefficients

EXAMPLE: $(x+2)^2 = (x+2)(x+2) = x(x) + x(2) + 2(x) + 2(2) = x^2 + 4x + 4.$
$(x+2)(x-2) = x(x) + x(-2) + 2(x) + 2(-2) = x^2 - 2^2 = x^2 - 4.$

NOTE: $(x+2)^2$ is NOT $x^2 + 2^2$.
ALSO: *Sum times Difference equals Difference of Squares*

Why is $(A+B)^2 \neq A^2 + B^2$?

Total area of box $= (A+B)^2$

$= A^2 + BA + AB + B^2$

$= (A+B)(A+B)$

NOTE: Given a sum of two numbers, their difference is called the **conjugate.** Similarly, given a difference, the sum is called the conjugate.
Example: $x+2$ is the conjugate of $x-2$. $\sqrt{3}-\sqrt{2}$ is the conjugate of $\sqrt{3}+\sqrt{2}$.

SOME PRACTICE PROBLEMS

ANSWERS AT THE BOTTOM

Simplify the following, as much as you can.

1. $(0.05c^3b + 0.02c^2b^2 - 0.09cb^3) - (-0.03c^3b + 0.08c^2b^2 - 0.1cb^3)$

2. $(2x - 5)(x + 4)$

3. $\left(\frac{1}{2}x^2 - x + 1\right)(x^3 + \frac{2}{3})$

4. $(3w^2 - 7z)(3w^2 + 7z)$

5. $(5m - 3)^2$

ANSWERS BELOW

SOME PRACTICE PROBLEMS

Simplify the following, as much as you can.

1. $(0.05c^3b + 0.02c^2b^2 - 0.09cb^3) - (-0.03c^3b + 0.08c^2b^2 - 0.1cb^3)$

$= (0.05 + 0.03)c^3b + (0.02 - 0.08)c^2b^2 + (-0.09 + 0.1)cb^3$

$= .08c^3b - .06c^2b^2 + .01cb^3$

2. $(2x - 5)(x + 4)$

$= (2x^2 + 8x - 5x - 20) = 2x^2 + 3x - 20$

3. $\left(\frac{1}{2}x^2 - x + 1\right)\left(x^3 + \frac{2}{3}\right)$

$= \frac{1}{2}x^5 + \frac{1}{2} \times \frac{2}{3}x^2 - x^4 - \frac{2}{3}x + x^3 + \frac{2}{3}$

$= \frac{1}{2}x^5 - x^4 + x^3 + \frac{1}{3}x^2 - \frac{2}{3}x + \frac{2}{3}$

4. $(3w^2 - 7z)(3w^2 + 7z)$

→ Sum of $3w^2$ & $7z$ times their difference

= Difference of squares

$= (3w^2)^2 - (7z)^2 = 9w^4 - 49z^2$

5. $(5m - 3)^2$

$= (A - B)^2 = A^2 - 2AB + B^2$

$= (5m)^2 - 2(5m)3 + 3^2$

$= 25m^2 - 30m + 9$

Multiplying term by term:

$(5m - 3)(5m - 3) = 25m^2 - 15m - 15m + 9$

$= 25m^2 - 30m + 9$