R3 Rational Exponents and Radicals

1-24-2018 class notes

Working with rational exponents / radicals

EXAMPLES

$\sqrt{2}$ is the number whose square equals 2.
So $(\sqrt{2})^2 = 2$.
We denote this another way by saying $\sqrt{2} = 2^{\frac{1}{2}}$.

Notice that according to rules of exponents,

$$(2^{\frac{1}{2}})^2 = 2^{\frac{1}{2}\times 2} = 2^1 = 2.$$

Similarly

$$\sqrt[3]{8} = 8^{\frac{1}{3}} = 2 \; and \; 2^3 = (\sqrt[3]{8})^3 = (8^{\frac{1}{3}})^3 = 8.$$

RATIONAL POWERS

You can also have other powers of the square-roots, cube-roots, etc.,

Examples

$$(\sqrt{2})^3 = (2^{\frac{1}{2}})^3 = 2^{\frac{3}{2}}.$$

$$(\sqrt[3]{8})^2 = (8^{\frac{1}{3}})^2 = 8^{\frac{1}{3}\times 2} = 8^{\frac{2}{3}} = 2^2 = 4$$

We can also have **negative radicals or fractional powers:**

$$4^{-\frac{1}{2}} = \frac{1}{4^{1/2}} = \frac{1}{2}.$$

Finally we can have combinations of these.
Note that the final answer is in radicals.
We can go back and forth between the two notations.

$$\frac{4^{1/4}2^{1/3}x^2}{2^{3/4}x^{3/2}} = \frac{(2^2)^{1/4}2^{1/3}x^2}{2^{3/4}x^{3/2}} = \frac{2^{1/2}2^{1/3}x^2}{2^{3/4}x^{3/2}}$$

$$= 2^{\frac{1}{2}+\frac{1}{3}-\frac{3}{4}}x^{2-\frac{3}{2}} = 2^{\frac{6+4-3\times 3}{12}}x^{\frac{1}{2}} = 2^{1/12}x^{1/2} = \sqrt[12]{2}\sqrt{x}.$$

By writing 81 as a power of 3, simplify $\sqrt[4]{81}$
81 equals 3^4. So $\sqrt[4]{81} = (3^4)^{1/4} = 3$.

By the way, $\sqrt{-4}$, $\sqrt[4]{-81}$ etc., DO NOT EXIST.
All even powers are positive,
so square root, fourth root, etc., work only for *positive* numbers.

SOME PRACTICE PROBLEMS.

Simplify each of the following.

Write answer in both radical and rational exponent notations.

$\sqrt[3]{-\frac{1}{8}}$ $= \sqrt[3]{\left(\frac{-1}{2}\right)^3} = \left(\left(-\frac{1}{2}\right)^3\right)^{1/3} = -\frac{1}{2}$

$16^{3/4} = \left(2^4\right)^{3/4} = 2^{4 \times 3/4} = 2^3 = 8$ Also $\left(= \left(\sqrt[4]{16}\right)^3\right.$

Write in radicals: $y^{4/11}$ $= \left(\sqrt[11]{y}\right)^4$

$\frac{a^{2/3}a^{5/3}}{a^{1/3}}$ $= \frac{a^{7/3}}{a^{1/3}} = a^{\frac{7}{3}-\frac{1}{3}} = a^2$ $\left(\text{Also } \frac{\sqrt[3]{a^2} \times \sqrt[3]{a^5}}{\sqrt[3]{a}} = \sqrt[3]{\frac{a^2 \times a^5}{a}} = \sqrt[3]{a^6}\right.$

$\sqrt[3]{250x^2y^6z^{11}}$ Note: $5^3 = 125 = \frac{250}{2}$

$= \sqrt[3]{5^3 \times 2 \times x^2 y^6 z^{11}} = 5 \times 2^{1/3} \times x^{2/3} y^{6/3} z^{\frac{11}{3}}$ $\left(\frac{11}{3} = 3\frac{2}{3}\right.$

$\left(\frac{3x^{1/2}}{y^{3/8}}\right)^4 \left(\frac{y^{1/2}}{3x^{4/3}}\right)^3$ $= 5 \times \sqrt[3]{2} \times \sqrt[3]{x^2} \times y^2 \times z^3 \times \left(\sqrt[3]{z^2}\right)$

$= \left(\frac{3^4 x^{4/2}}{y^{12/8}}\right)\left(\frac{y^{3/2}}{3^3 x^4}\right)$ [Multiply all powers by 4 4 3

$= \frac{3^4}{3^3} \times \frac{x^2}{x^4} \times \frac{y^{3/2}}{y^{3/2}} = \frac{3}{x^2} = 3x^{-2}$

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